AN EXPERIMENTAL INVESTIGATION OF A LOW DISTORTION MIXER USING A BEAM-DEFLECTION TUBE

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AN EXPERIMENTAL INVESTIGATION OF A LOW DISTORTION MIXER

USING A BEAM-DEFLECTION TUBE

Approved:

Date Approved by Chairman: May 29, 1961

PREFACE

This study is an experimental investigation of a low distortion mixer using a beam-deflection tube as the active circuit element. The material herein is limited in its scope in that the investigation covers only one of several basic circuit configurations that are suitable for use with beam-deflection tubes. It is also limited because only one type of beam-deflection tube has been considered.

The purpose of this investigation was to design a low distortion mixer for use in receivers such that distortion products generated in the translational portion of receivers can be made small enough in comparison to the desired signal so that the potential advantage of Single Sideband techniques will not be limited by the receiving equipment used.

A great deal of work has been carried out by researchers toward the development of low distortion power amplifiers for use in Single Sideband systems, but relatively little work aimed at producing low distortion mixers has been reported. Conventional mixers used in communications receivers possess several disadvantages which are covered in Chapter II of this work. These disadvantages are at least partially overcome by the use of a well designed beam-deflection tube as the active circuit element in properly designed mixers.

I gratefully acknowledge the assistance of the members of my reading committee, Professor Howard McKennley and Mr. W. B. Warren, whose advice, both technically and editorially, have made my problems in this work a pleasure. The aid, advice, and counsel of my thesis advisor, Dr. D. L.

Finn, which has been a source of encouragement to my throughout this entire endeavor, is also gratefully acknowledged. To my wife Carolyn, to whom this work is dedicated, I wish to express my deepest appreciation for her endless patience and encouragement.

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GLOSSARY OF TERMS

E_{bb} = plate supply voltage

E_{g2} = screen grid supply voltage

E_{g1} = control grid bias voltage

E_{dL} = dc potential of the left-hand deflection electrode

E_{dR} = dc potential of the right-hand deflection electrode

e_{sd} = ac voltage applied to the right-hand deflection electrode

e_{sg1} = ac voltage applied to the control grid.

SUMMARY

Intermodulation distortion, particularly third and fifth order, if generated in any part of a single sideband system can be detrimental. One portion of such systems which has received comparatively little attention by researchers is the mixer used in receiving equipment. A recently developed beam-deflection tube was investigated as an active element for use in a low distortion mixer. Such a low distortion mixer was investigated both analytically and experimentally, and the data relating the intermodulation characteristics of the mixer to the mixer parameters are presented both in tabular and in graphical form. Appropriate conclusions and comparisons regarding the usefulness and operations of such a mixer are given.

The experimental investigation reveals that, with the proper choice of mixer parameters, mixers having intermodulation products with levels 60 db below the level of the desired signal can be realized.

CHAPTER I

INTRODUCTION

With the advent of Single Sideband (SSB) techniques several years ago, many new communications problems arose. Most of these problems have been satisfactorily solved, but certain of them have received little attention. Some of the problems that have not received sufficient attention are the development of adequate AGC and AVC receiver circuitry, power amplifier distortion monitoring equipment (1), and low distortion mixers for use in both transmitting and receiving equipment. In this study an attempt has been made to present a possible solution to the latter of these mentioned problems.

One of the most stringent operating conditions of a Single Sideband system is the commercial or military use of SSB for multiplex operation wherein several different voice communication channels are transmitted in each sideband. In such a system the presence of intermodulation (IM) distortion products result in undesired interference which is manifested in the form of inter-channel crosstalk. Crosstalk is a form of "babble" and is a most annoying form of interference (2, 3). Thusly, the concern of the system design engineer can not be limited to the linearity of the transmitting equipment alone, for the generation of intermodulation products in any part of the system will result in crosstalk and a degradation of the full potential of the SSB technique.

The state of the art in the design of linear power amplifiers is such that the intermodulation products generated in the r-f portions of SSB transmitters can be kept 50 db below the level of the desired signal (4). It is thus apparent that, if the mixers used in either the transmitting or receiving equipment generate distortion products which are of any greater level, relative to desired signal, then those generated in the power amplifiers, the entire system performance suffers, and only by reduction of distortion in all portions of the SSB system can the state of the art be advanced.

CHAPTER II

INTERMODULATION PRODUCTS AND THEIR GENERATION

The use of single sideband techniques as opposed to amplitude modulation, frequency modulation, pulse code modulation and others, provides several advantages. Possibly the greatest advantage that single sideband has over these other forms of communications is the conservation of the radio frequency spectrum. As the name "single sideband" indicates, the transmitted signal is limited to either the upper or the lower sideband of the generated signal. This is usually accomplished in either of two ways. Either the signal is generated entirely by electronic means using balanced modulators and phase shifting networks or a double sideband zero carrier (DSBZC) signal is generated using a balanced modulator and the undesired sideband is filtered out by use of a high attenuation filter having the necessary and desired shape factor. Regardless of how the signal is generated only one sideband is transmitted. If an audio signal such as is shown in Equation (1) is applied to a balanced modulator to which a carrier, such as is shown in Equation (2), is applied, the resulting output will be a signal as is given in Equation (3).

$$f(t) = A \cos \omega_1 t + A \cos \omega_2 t \tag{1}$$

$$F_c(t) = B \cos \omega_c t \tag{2}$$

$$\begin{aligned} F_{dsb}(t) = {} & C \cos (\omega_c - \omega_1)t + C \cos (\omega_c - \omega_2)t + \\ & C \cos (\omega_c + \omega_1)\, t + C \cos (\omega_c + \omega_2)t \end{aligned} \tag{3}$$

If for example the lower sideband of the signal given in Equation (3) is to be transmitted, the upper sideband must be removed. One method to accomplish this is the filter method mentioned above. If the upper sideband is filter out the resulting signal will be

$$F_{ssb}(t) = C \cos (W_c - W_1)t + C \cos (W_c - W_2)t \ . \tag{4}$$

This signal consists of two spectral spikes located in the frequency spectrum at a frequency that is the difference between the carrier frequency and the frequencies of the components of the modulating signal.

Before such a signal as this is applied to an antenna it is necessary to amplify it to a point such that resulting power output of the transmitter is sufficient to maintain some minimum signal to noise ratio at the receiver. In order to increase the power level of this signal it is applied to a linear amplifier. The design of such linear amplifiers has been considered important, as is witnessed by the great deal of work that researchers have done in this field (4, 5, 6). The great majority of this work has been directed at the problem of distortion present in "Linear Amplifiers" and methods of reducing this distortion. In this connection the most troublesome forms of distortion are the "in-band" or "odd-order" intermodulation products. "In-band" or "odd-order" intermodulation products are a special case of the general intermodulation product which we shall now examine.

Intermodulation products are spurious signals arising from the interaction between the components of the original input signal. When a signal such as the signal given in Equation (4) is applied to some nonlinear device new frequencies will be generated. If the output current of this nonlinear device can be expressed in the form of a series as is given in Equation (5),

$$i_p(t) = a_o + a_1F_{ssb}(t) + a_2F_{ssb}(t)^2 + a_3F_{ssb}(t)^3 + \ldots\ldots a_nF_{ssb}(t)^n + \ldots \quad (5)$$

new frequency components will appear at the output. Among the various newly generated frequency components, there will be several of the form

$$f_3(t) = D \cos \left[N(\omega_c - \omega_1) - M(\omega_c - \omega_2) \right] t,$$

where M and N are integers. If we consider such components and restrict our attention to values of N = 2, 3, 4, 5, ... etc., and values of M = N-1, we will be considering the components commonly referred to as the "odd-order" or "in-band" intermodulation products.[1] For values of N which are sufficiently small the distortion product will occur near the original signal frequency, and in the case of multiplex operation such products can cause the undesired crosstalk type of interference.

If it is desired to transmit four channels of information, for example, each of which consists of a single tone signal, in the lower sideband of a single sideband signal, the transmitted signal can be written as in Equation (6).

$$f_4(t) = A \cos (\omega_c - \omega_1)t + B \cos (\omega_c - \omega_2)t + C \cos (\omega_c - \omega_3)t \quad (6)$$
$$+ D \cos (\omega_c - \omega_4)t$$

where ω_1, ω_2, ω_3, and ω_4 are the frequencies of the original four tones, and ω_c is the carrier frequency of the SSB signal. If this signal is

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[1]This derivation is shown in detail in Appendix I, page 45.

passed through an amplifier or any other device which has nonlinearities, intermodulation products will be generated. Furthermore if the order of curvature of the nonlinear device is of the correct nature "odd-order" or "in-band" intermodulation products will also be present. The spectral distribution of the original signal is shown in Figure 1, and a possible spectral distribution of the distorted signal is shown in Figure 2. The inband distortion products shown in this figure give an intuitive yet immediate feeling for the possible effects of distortion present in a single sideband system.

Though the above discussion refers primarily to transmitters and linear amplifiers, the same effects are present no matter where in the SSB system the nonlinearities occur, and in particular if the nonlinearities occur in the translational portion of the communications receiver at the termination of the circuit, the nonlinearities will be generated and will be just as detrimental as if they were generated in the transmitter.

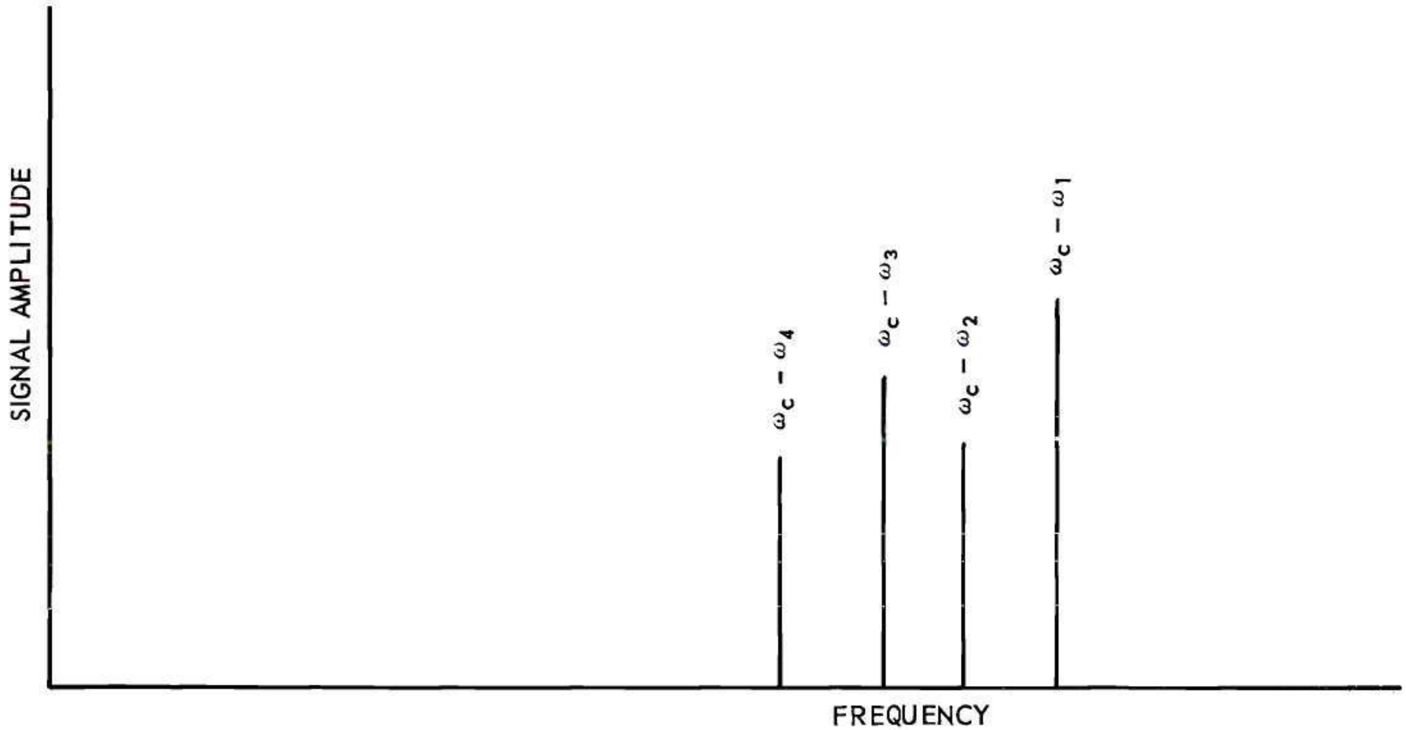

Figure 1. Spectral Distraction of Clean Four-Tone SSB Signal.

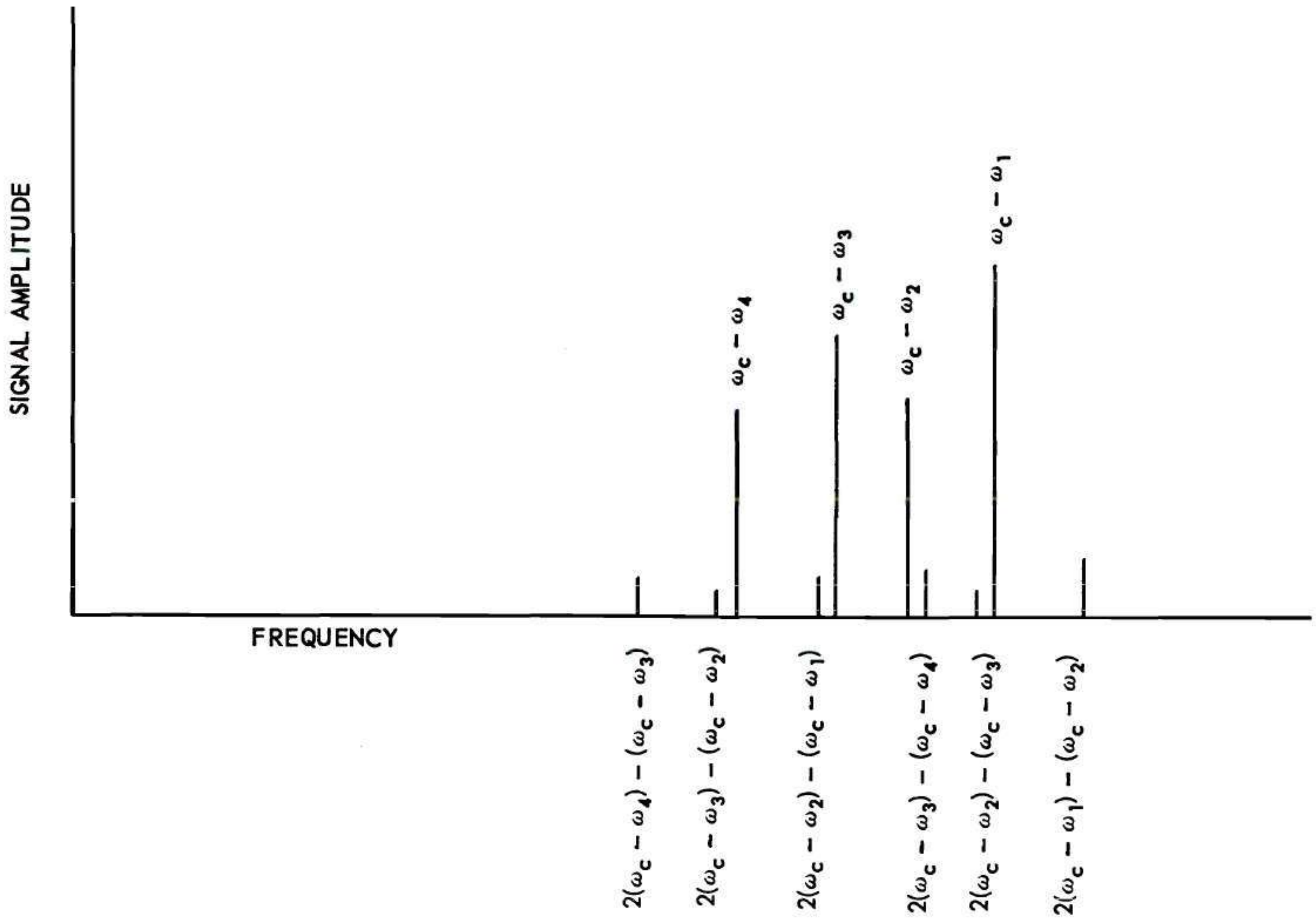

Figure 2. Possible Spectral Distribution of Distorted Four-Tone SSB Signal.

CHAPTER III

CONVENTIONAL MIXERS

A mixer is a device that translates a signal from one part of the radio spectrum to another part of the spectrum. In the case of communications receivers the mixer is used so that no matter what the frequency of the received signal may be, it is always translated to some common frequency so as to reduce the number of stages of the receiver that must be capable of tracking. A variety of vacuum-tube arrangements find practical use as mixers. One of the most commonly encountered is the suppressor-grid-modulated arrangement. A typical circuit diagram of such a mixer is shown in Figure 3.

The conventional analysis of the circuit shown in Figure 3 is made by assuming that the total plate current of the pentode is related to the instantaneous sum of the input signal and the local oscillator signal by some transfer function. In the conventional analysis, a single tone input signal is usually assumed. Thus the problem of intermodulation product generation is ignored. However, if we assume some reasonable transfer function for the tube in the circuit of Figure 3 and apply a similar analysis as was used in Chapter I, it will be found that intermodulation products are present in the output signal. Assume the input signal to the mixer, e_s, is a clean two tone test signal as in Equation (7)

$$e_s = A \cos \omega_1 t + A \cos \omega_2 t \qquad (7)$$

and that the local oscillator signal is a single tone as in Equation (8).

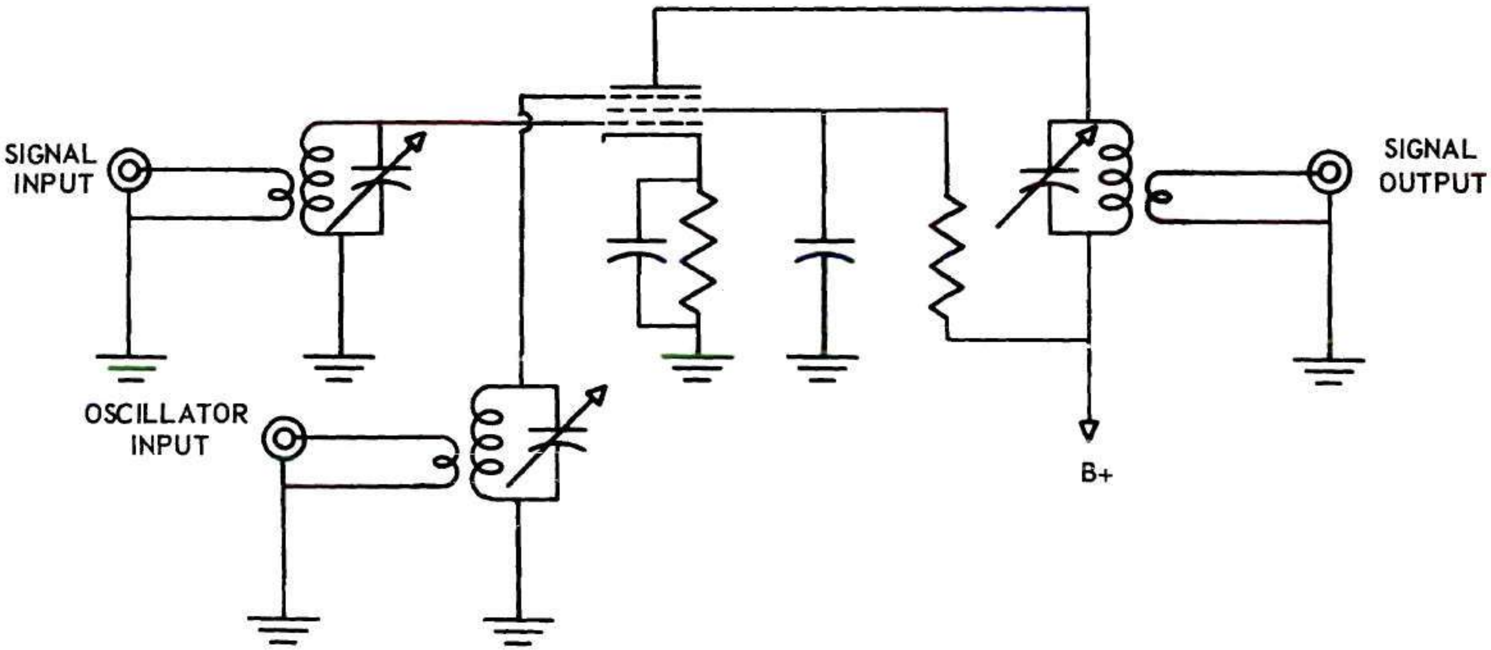

Figure 3. Typical Suppressor-Grid-Modulated Mixer.

$$e_{LO} = B \cos \omega_c t \tag{8}$$

Further assume that the following series, Equation (9), adequately represents the tube's transfer function

$$\begin{aligned} f_r = {} & a_0 + a_1 e_g + a_2 e_g^2 + a_3 e_g^3 + a_4 e_g^4 + a_5 e_g^5 + \ldots. \\ & + a_N e_g^N + \ldots\ldots. \end{aligned} \tag{9}$$

where

$$e_g = e_s + e_{LO} \, . \tag{10}$$

Then the plate current of the pentode will be

$$\begin{aligned} i_p = {} & a_0 + a_1(e_s + e_{LO}) + a_2(e_s + e_{LO})^2 + a_3(e_s + e_{LO})^3 \\ & + a_4(e_s + e_{LO})^4 + a_5(e_s + e_{LO})^5 + \ldots. \\ & + a_N(e_s + e_{LO})^N + \ldots\ldots \end{aligned} \tag{11}$$

$$\begin{aligned} i_p = {} & a_0 + a_1 e_s + a_1 e_{LO} + a_2 \left[e_s^2 + 2 e_s e_{LO} + e_{LO}^2 \right] \\ & + a_3 \left[e_s^3 + 3 e_s^2 e_{LO} + 3 e_s e_{LO}^2 + e_{LO}^3 \right] \\ & + a_4 \left[e_s^4 + 4 e_s^3 e_{LO} + 6 e_s^2 e_{LO}^2 + 4 e_s e_{LO}^3 + e_{LO}^4 \right] \\ & + a_5 \left[e_s^5 + 5 e_s^4 e_{LO} + 10 e_s^3 e_{LO}^2 + 10 e_s^2 e_{LO}^3 + \right. \\ & \left. 5 e_s e_{LO}^4 + e_{LO}^5 \right] + \ldots\ldots + a_N \left[e_s^N + (N) e_s^{N-1} e_{LO} \right. \\ & \left. + \ldots. + (N)\, e_s e_{LO}^{N-1} + e_{LO}^N \right] + \ldots \end{aligned} \tag{12}$$

Substituting Equation (7) and Equation (8) into Equation (12) gives

$$
\begin{aligned}
i_p = a_0 &+ a_1 A \cos \omega_1 t + a_1 A \cos \omega_2 t + a_1 B \cos \omega_c t \qquad (13) \\
&+ a_2 A^2 \cos^2 \omega_1 t + a_2 A^2 \cos^2 \omega_2 t + 2a\ A^2 \cos \omega_1 t \cos \omega_2 t \\
&+ 2a_2 A\ B \cos \omega_1 t \cos \omega_c t + 2a_2 A\ B \cos \omega_2 t \cos \omega_c t \\
&+ a_2 B^2 \cos^2 \omega_c t + a\ A^3 \cos^3 \omega_1 t + 3a\ A^3 \cos^2 \omega_2 t \cos \omega_1 t \\
&+ 3a_3 A^3 \cos^2 \omega_1 t \cos \omega_2 t + a_3 A^3 \cos^3 \omega_2 t + \\
&+ 3a_3 A^2\ B \cos^2 \omega_1 t \cos \omega_c t + 3a_3 A^2\ B \cos^2 \omega_2 t \cos \omega_c t \\
&+ 6a_3 A^2\ B \cos \omega_1 t \cos \omega_2 t \cos \omega_c t + 3a_3 A\ B^2 \cos \omega_1 t \cos^2 \omega_c t \\
&+ 3a_3 A\ B^2 \cos \omega_2 t \cos^2 \omega_c t + a_3\ B^3 \cos^3 \omega_c t + a_4 A^4 \cos^4 \omega_1 t \\
&+ 4a_4 A^4 \cos^3 \omega_1 t \cos \omega_2 t + 6a_4 A^4 \cos^2 \omega_1 t \cos^2 \omega_2 t \\
&+ 4a_4 A^4 \cos \omega_1 t \cos^3 \omega_2 t + a_4 A^4 \cos^4 \omega_2 t \\
&+ 4a_4 A^3\ B \cos^3 \omega_1 t \cos \omega_c t + 12a_4 A^3 B \cos^2 \omega_1 t \cos \omega_2 t \cos \omega_c t \\
&+ 12a_4 A^3 B \cos \omega_1 t \cos^2 \omega_2 t \cos \omega_c t + 4a_4 A^3 B \cos^3 \omega_2 t \cos \omega_c t \\
&+ 6a_4 A^2 B^2 \cos^2 \omega_1 t \cos^2 \omega_c t + 12a_4 A^2 B^2 \cos \omega_1 t \cos \omega_2 t \cos^2 \omega_c t \\
&+ 6a_4 A^2 B^2 \cos^2 \omega_2 t \cos^2 \omega_c t + a_4 A\ B^3 \cos \omega_1 t \cos^3 \omega_c t \\
&+ 4a_4 A B^3 \cos \omega_2 t \cos^3 \omega_c t + a_4 B^4 \cos^4 \omega_c t + a_5 A^5 \cos^5 \omega_1 t \\
&+ 5a_5 A^5 \cos^4 \omega_1 t \cos \omega_2 t + 10a_5 A^5 \cos^3 \omega_1 t \cos^2 \omega_2 t \\
&+ 10a_5 A^5 \cos^2 \omega_1 t \cos^3 \omega_2 t + 5a_5 A^5 \cos \omega_1 t \cos^4 \omega_2 t
\end{aligned}
$$

$$+ a_5A^5 \cos^5\omega_2 t + 5a_5A^5 \cos^4\omega_1 t \cos \omega_2 t + 10a_5A^5 \cos^3\omega_1 t \cos^2\omega_2 t$$

$$+ 10a_5A^5 \cos^2 \omega_1 t \cos^3 \omega_2 t + 5a_5A^5 \cos \omega_1 t \cos^4 \omega_2 t$$

$$+ 10a_5A^5 \cos^2\omega_1 t \cos^3 \omega_2 t + 5a_5A^5 \cos \omega_1 t \cos^4 \omega_2 t$$

$$+ a_5A^5 \cos \omega_2 t + 5a_5A^4B \cos^4 \omega_1 t \cos \omega_c t$$

$$+ 20\, a_5A^4B \cos^3 \omega_1 t \cos \omega_2 t \cos \omega_c t$$

$$+ 35\, a_5A^4B \cos^2 \omega_1 t \cos \omega_2 t \cos \omega_c t$$

$$+ 20\, a_5A^4B \cos \omega_1 t \cos^3 \omega_2 t \cos \omega_c t + 5a_5\, A^4B \cos^4\omega_2 t \cos \omega_c t$$

$$+ 10a_5A^3B^2 \cos^3 \omega_1 t \cos^2 \omega_c t + 30a_5A^3B^2 \cos^2\omega_1 t \cos \omega_2 t \cos^2\omega_c t$$

$$+ 30\, a_5A^3B^2 \cos \omega_1 t \cos^2\omega_2 t \cos^2 \omega_c t$$

$$+ 10a_5A^3B^2 \cos^3\omega_2 t \cos^2\omega_c t + 10a_5A^2B^3\cos^2 \omega_1 t \cos^3 \omega_c t$$

$$+ 20\, a_5A^2B^3 \cos \omega_1 t \cos^3\omega_2 t \cos \omega_c t + 10a_5A^2B^3 \cos^2\omega_2 t \cos^3\omega_c t$$

$$+ 5a_5AB^4 \cos \omega_1 t \cos^4 \omega_c t + 5a_5\, AB^4 \cos \omega_2 t \cos^4 \omega_c t$$

$$+ B^5 \cos^5 \omega_c t + \text{contributions from higher order terms.}$$

Expanding Equation (13) and considering only the components having a frequency on or near the difference frequency arising from fifth and lower order curvature we have,

$$i_{p_{IF}} = K_1 \cos (\omega_c - \omega_1)t + K_2 \cos (\omega_c - \omega_2)t \tag{14}$$

$$+ K_3 \cos \left[\omega_c - (2\omega_2 - \omega_1)t\right] + K_4 \cos \left[\omega_c - (2\omega_1 - \omega_2)t\right]$$

where the difference frequency is $\omega_{IF} = \omega_c - \dfrac{\omega_1 + \omega_2}{2}$, and

$$K_1 = K_2 = a_2AB + 7a_4A^3B + \frac{3}{2}a_4 \; A \; B^3, \quad K_3 = K_4 = \frac{3a_4A^3B}{2} \quad .$$

When the analysis is carried out for a sufficient number of terms, as is done above, the intermodulation products appear, and as can be seen from Equation (14) they will be significant if the coefficients are large.

In presently used practical mixers, these coefficients are such that most SSB systems have an IM product level only 30 db below the level of the desired signal. In some specific cases[1,4] mixers using conventional techniques have been designed having low distortion. In these cases, however, the conversion amplification of the mixer, the ratio of the output signal amplitude at the intermediate signal frequency to the input signal amplitude at the input signal frequency, is sacrificed in order to obtain the low levels of distortion. The figure of conversion amplification in such mixers is usually less than unity; this is an undesirable feature in most cases.

On the other extreme mixers having good conversion amplification figures can be designed, but these mixers are characterized by distortion products which are only about 20 - 30 db below the level of the desired signal. It is possible to design a mixer, using a recently developed beam deflection tube, which overcomes both the disadvantages of low gain and high distortion generation. The mathematical analysis of such a mixer is given in Chapter IV and the experimental investigation is outlined in Chapter V.

CHAPTER IV

A LOW DISTORTION MIXER USING A BEAM DEFLECTION TUBE

The transfer function of a pentode vacuum tube can be approximated in the form of a series as in Equation (15).

$$i_0 = a_0 + a_1 e + a_2 e^2 + a_3 e^3 + \ldots\ldots a_N e^N + \ldots\ldots \tag{15}$$

where i_0 = instantaneous plate current

e = instantaneous grid voltage, and

a_i = the coefficient of the i^{th} order curvature of the tube's transfer function.

It is assumed that variations of the plate voltage of a pentode have negligible effect on the plate current. In a great many applications, such as linear amplifiers, it is desirable to have as little second and higher order curvature as possible in order to minimize distortion of the signal. In other applications, such as mixers, certain types of nonlinearities are necessary in order that fundamental mixing takes place, but excessive nonlinearities lead to the distortion of the signal being translated. In the case of the mixer it is desirable that nonlinearities which distort the signal be minimized while nonlinearities that contribute to the mixing process be maintained. In a conventional mixer the input signal can be considered to be made up of the signal to be translated plus the local oscillator signal and the resulting output can be calculated if the coefficients of the series representation of the mixer tube's transfer function are known. In order that the output signal not be appreciably distorted, several conditions must be met in the conventional mixer. Among them the most significant apparently are:

1. The tube bias must be properly set and maintained within close tolerances.
2. The input signal must usually be small in amplitude.
3. The amplitude of the oscillator injection voltage must be a specific value, depending on the tube and the bias voltage, and must be maintained within close tolerances.

If these conditions are satisfied the resulting distortion of the signal can be quite small, depending on the choice of tubes. However, it should be pointed out that the proper conditions for low distortion operation vary a great deal from tube to tube of the same type and same manufacturer. It should also be pointed out that as aging of the oscillator tube occurs the injection voltage to the mixer usually decreases causing the signal transfer function to become more nonlinear. This increase in distortion is also accompanied by a marked decrease in the conversion amplification due to the decrease in the conversion transconductance. The conversion amplification is defined as the ratio of the amplitude of the output signal, at the difference frequency, to the amplitude of the input signal at the input signal frequency.

A beam-deflection tube is a multielement tube and its construction is shown schematically in Figure 4. The electron gun, consisting of the filament, the cathode, the control grid, and the screen grid or accelerator, establishes an electron beam and accelerates the electrons of the beam toward the anodes of the tube. Upon passing through a beamwidth limiting

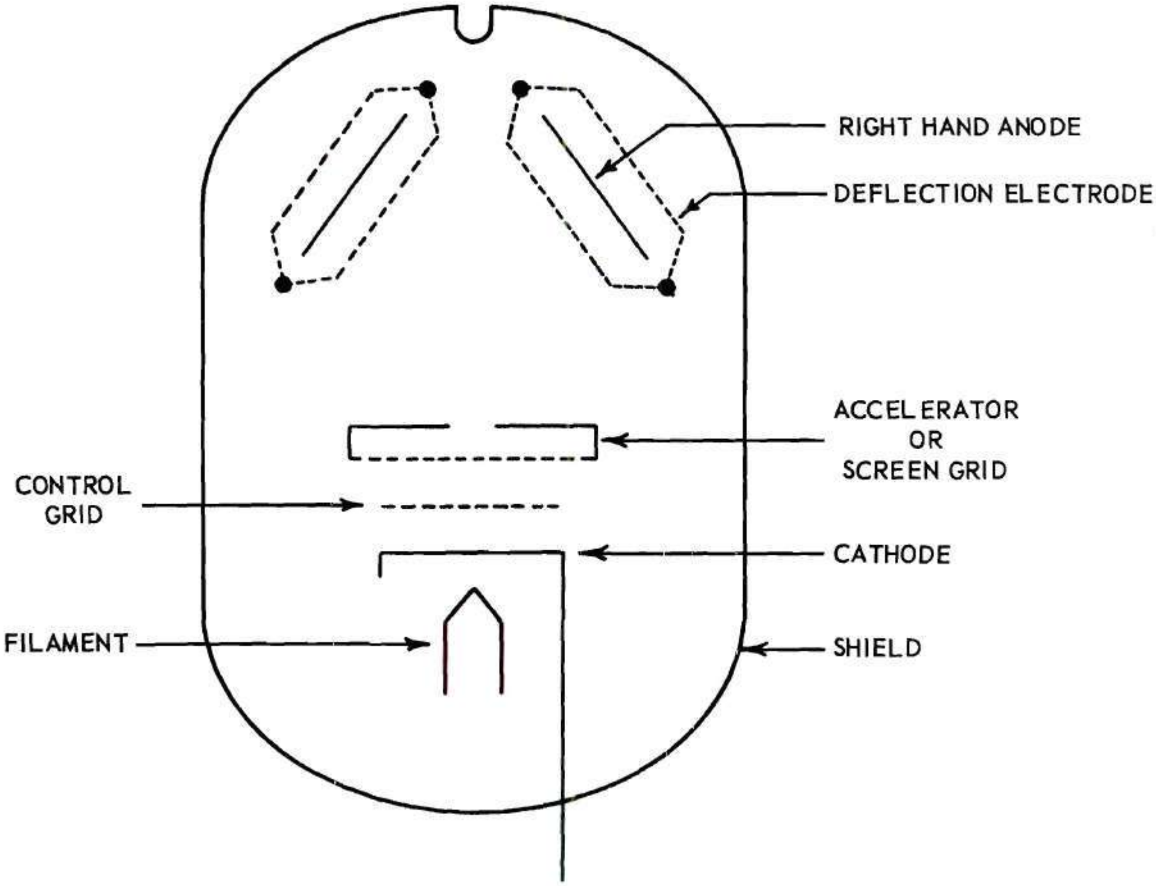

Figure 4. Simplified Internal Construction of RCA 7360
Beam-Deflection Tube.

slit in the accelerator, the electrons are subjected to a deflecting field that is present when there is a potential difference between the two deflection electrodes. As a result of this field the beam is deflected from its original path. As a result of this deflection more electrons reach one anode than the other. In a beam-deflection tube, similar to the case of the pentode, the total beam current can be closely approximated by a series as is given in Equation (16), and the current to one plate can be represented by another series that has the total beam current as a factor as is shown in Equation (17).

$$i_0 = a_0 + a_1 e_{LO} + a_2 e_{LO}^2 + a_3 e_{LO}^3 + \ldots\ldots a_N e_{LO}^N + \ldots\ldots \tag{16}$$

$$i_L = i_0 \left[b_0 + b_1 e_s + b_2 e_s^2 + \ldots\ldots b_N e_s^N + \ldots \right] \tag{17}$$

where i_0 is the total beam current, e_{LO} is the local oscillator voltage applied to the control grid, i_L is the current in the left hand plate, a_i and b_i are the i^{th} coefficients associated with the control grid and the deflection electrodes, respectively, and e_s is the signal to be translated which is applied to the deflection electrodes. The current flowing in the right hand plate circuit, i_R, will be the total beam current minus the current flowing in the left hand plate circuit. This may be written as

$$i_R = i_0 - i_L \,. \tag{18}$$

If the plate circuit is arranged in a push-pull manner, the output voltage will be proportional to the current flowing in the primary and this current will be the difference of the currents flowing in the two plates. This difference current, i_D, may be written as follows:

$$i_D = i_R - i_L = (i_0 - i_L) - i_L = i_0 - 2i_L \tag{19}$$

Substituting Equation (17) into Equation (19) yields:

$$i_D = i_0 - 2i_0\left[b_0 + b_1e_s + b_2e_s^{\,2} + \ldots\ldots b_Ne_s^{\,N} + \ldots\ldots\right] \tag{20}$$

Equation (20) may be rewritten as

$$i_D = i_0\left[1 - 2b_0 - 2b_1e_s - 2b_2e_s^{\,2} \ldots\ldots -2b_Ne_s^{\,N} + \ldots\ldots\right] \tag{21}$$

Letting

$$\begin{aligned} 1 - 2b_0 &= B_0\,, \\ -2b_1 &= B_1\,, \\ -2b_2 &= B_2\,, \end{aligned} \tag{22}$$

Equation (21) becomes:

$$i_D = i_0\left[B_0 + B_1e_s + B_2e_s^{\,2} + B_3e_s^{\,3} + \ldots B_Ne_s^{\,N} + \ldots\ldots\right] \tag{23}$$

Substituting Equation (16) into Equation (23), we have

$$i_D = \left[a_0 + a_1e_{LO} + a_2e_{LO}^{\,2} + \ldots\right]\left[B_0 + B_1e_s + B_2e_s^{\,2} + \ldots\right] \tag{24}$$

For small signal input, the higher order coefficients of Equation (23) may be neglected. Thus Equation (24) may be reduced to the following form:

$$i_D = \left[a_0 + a_1e_{LO} + a_2e_{LO}^{\,2} + \ldots\right]\left[B_0 + B_1e_2\right] \tag{25}$$

$$= a_0B_0 + a_1B_0e_{LO} + a_2B_0e_{LO}^{\,2} + a_0B_1e_s + a_1B_1e_se_{LO} + a_2B_1e_se_{LO}^{\,2} \ldots \tag{26}$$

Collecting and redefinine coefficients, Equation (26) may be written as follows:

$$i_D = C_0 + C_1 e_{LO} + C_2 e_{LO}^2 + D_0 e_s + D_1 e_s e_{LO} + D_2 e_s e_{LO}^2 + \ldots C_N e_{LO}^N \qquad (27)$$

$$+ D_N e_s e_{LO}^N \ldots\ldots$$

where

$$C_0 = a_0 B_0 \qquad (28)$$

$$C_1 = a_1 B_0$$

$$C_2 = a_2 B_0$$

$$D_1 = a_0 B_1$$

$$D_2 = a_1 B_1$$

etc.

Now consider the expansion of equation 27[1] in which

$$e_{LO} = A \cos \omega_1 t \qquad (29)$$

and[2]

$$e_s = B \cos \omega_2 t + B \cos \omega_3 t \;, \qquad (30)$$

and assume that the output is coupled through a resonant transformer which is tuned to a frequency that is the difference between the mean input signal frequency, $\frac{\omega_2 + \omega_3}{2}$, and the local oscillator signal frequency. It can be shown that the resulting equation is,

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[1]This derivation is shown in Appendix I, page 50.

[2]It has been found that a two tone signal of the type shown in Equation (30) adequately simulates single sideband operation. (See Bruene: Distortion Reducing Means for Single-Sideband Transmitters, Proceedings of the IRE, December 1956, page 1760.)

$$i_D = \left[AD_2B/2 + 3A^3D_4B/8 + 10A^5D_6B/32 + \ldots\ldots\right] \cos\,(\omega_2 - \omega_1)t \quad (31)$$
$$+ \left[AD_2B/2 + 3A^3D_4B/8 + 10A^5D_6B/32 + \ldots\ldots\right] \cos\,(\omega_3 - \omega_1)t$$

and the output voltage is given by

$$V_0 = Ki_D \quad (32)$$

Intermodulation products of the third order, would appear in a form such as follows:

$$i_{IM} = \cos\left[(2\omega_2 - \omega_3) - \omega_1\right]t \quad \text{or} \quad (33)$$
$$i_{IM} = \cos\left[(2\omega_3 - \omega_2) - \omega_1\right]t$$

It can be seen from Equation (31) that there is no third order intermodulation distortion generated in the mixer if the foregoing assumptions are correct. Similarly, no higher order distortion products are generated. Because there are no intermodulation products present, the magnitude of the oscillator injection voltage, A, does not effect the amount of distortion present. This is not the case in the conventional mixer where changes in the oscillator injection voltage appreciably effect the distortion generated. Now in fact, there are higher order terms than have been indicated for the transfer function of the input signal, but the resulting intermodulation products generated are small. Thus, it is possible, by use of a high quality beam-deflection tube, to construct a mixer having good intermodulation rejection characteristics and which has no appreciable increasing intermodulation products generated when the oscillator injection voltage decreases. The experimental investigation necessary to validate this analysis is presented in Chapter V, and conclusions relating the theory to practice is presented in Chapter VI.

CHAPTER V

THE EXPERIMENTAL ARRANGEMENT AND DESCRIPTION OF CONDUCTED TESTS

A mixer such as has been described in Chapter IV was designed and constructed. The circuit was arranged so that the important mixer parameters, i.e., control grid bias, plate supply voltage, plate load impedance, and deflection electrode potential could be varied. In addition to these variables the levels of both the input signal and the local oscillator signal could be varied. The detailed circuit diagram of the mixer is shown in Figure 5 and a photograph of the mixer is shown in Figure 6. The remaining portion of the equipment required for the experimental investigation is shown in the block diagram of Figure 7 and in the photographs of Figure 8.

In order to obtain a clean two-tone test signal two separate generators were used. The signal from each generator was low pass filtered so as to reduce the level of the generator harmonics. The filters were designed and constructed so as to have a maximum attenuation at the second harmonic of the generator frequency. After filtering, both signals were injected into a hybrid ring and the output of the hybrid ring was used as the two tone test signal. The purpose of the hybrid ring is to minimize the coupling of one generator signal into the output stage of the other generator. Such steps were found to be necessary in order to obtain a test signal which was itself effectively free from intermodulation products. Two photographs of the display of the spectrum analyzer showing the cleanliness of the two tone test signal are shown in Figure 9. The third order intermodulation product content

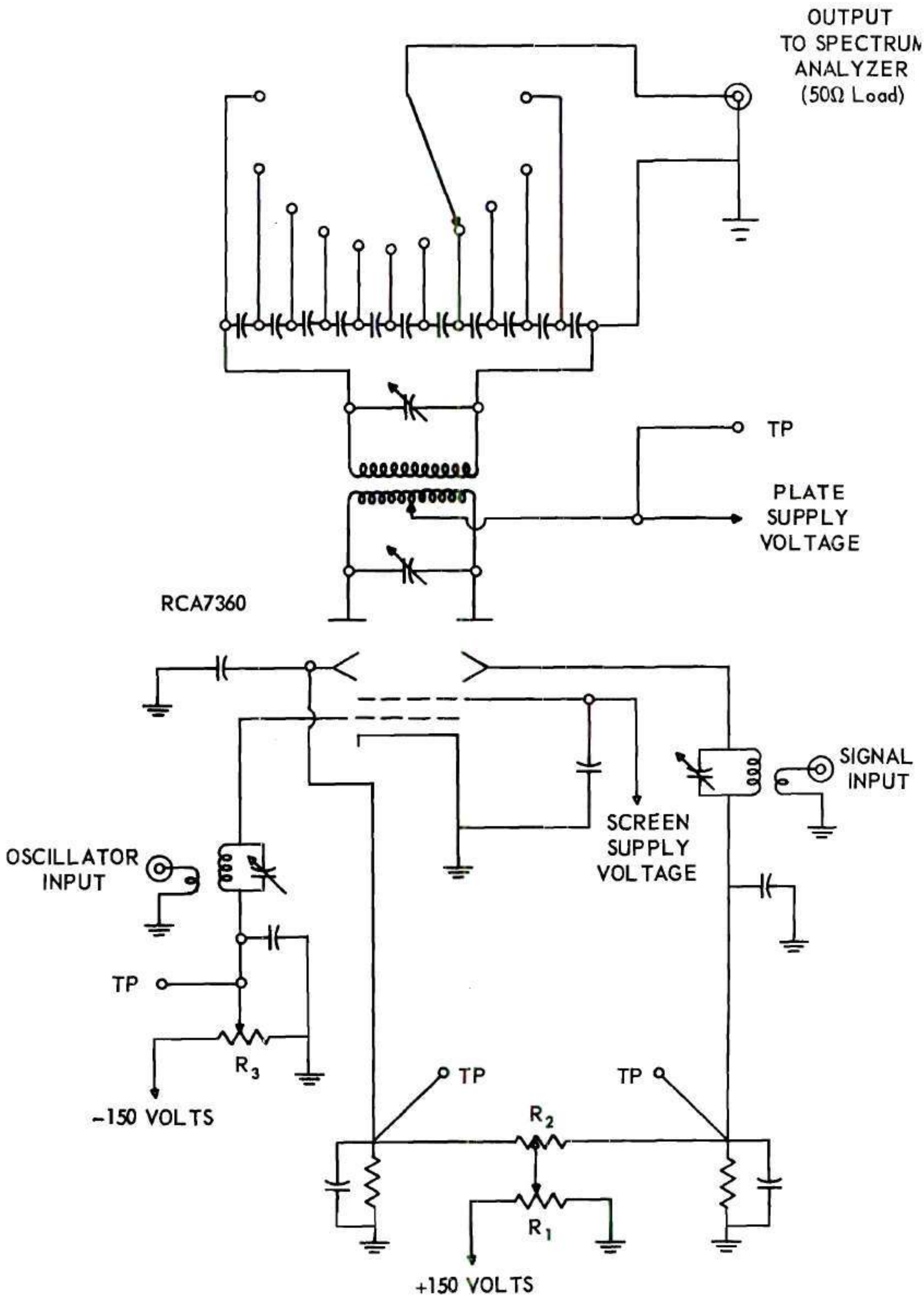


Figure 5. Detailed Circuit Diagram of Investigated Mixer.

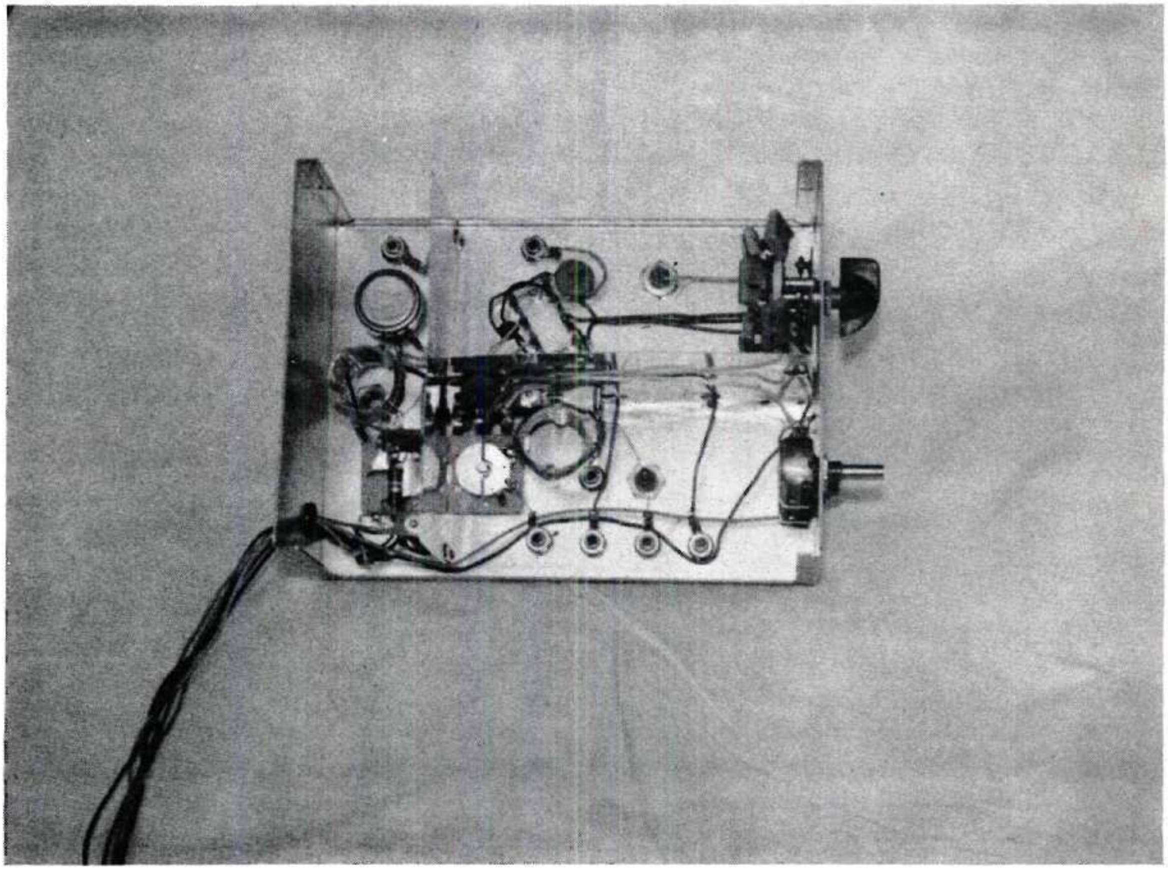


Figure 6. Internal View of Mixer Construction.

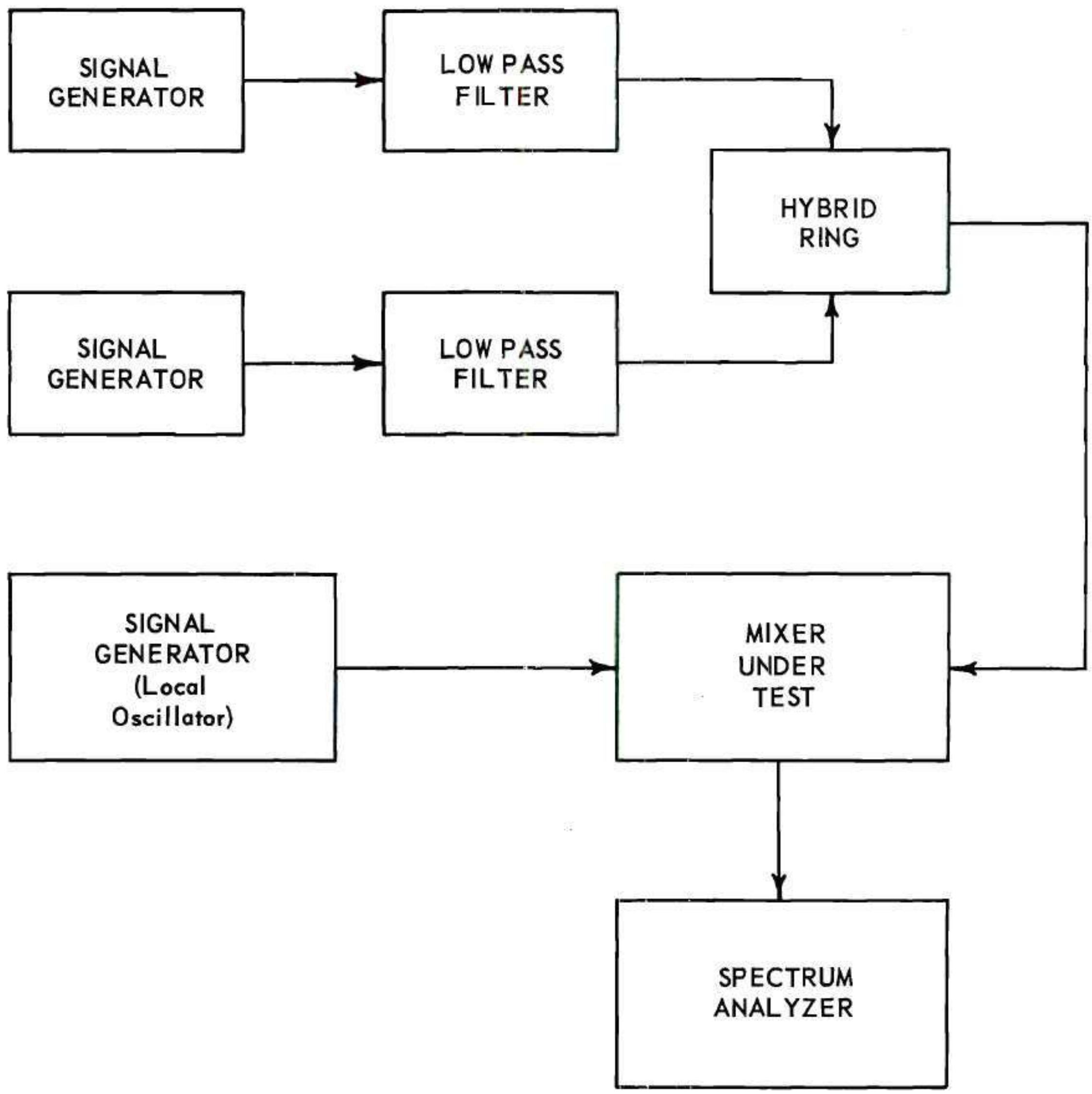
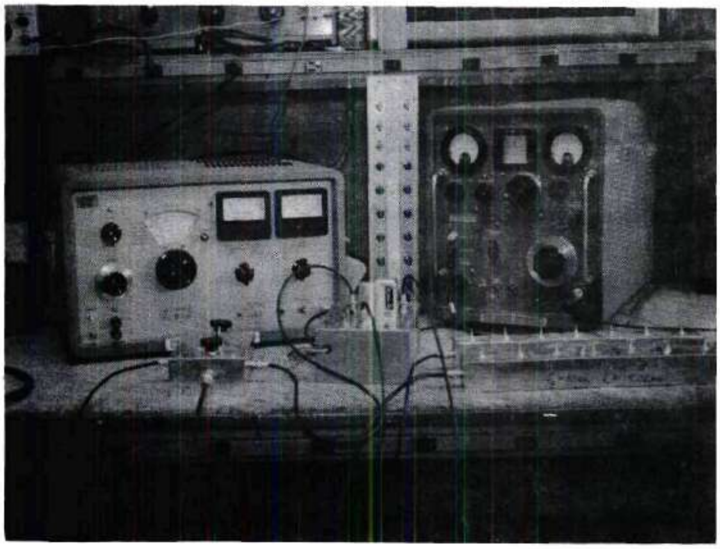


Figure 7. Block Diagram of Experimental Set-Up.



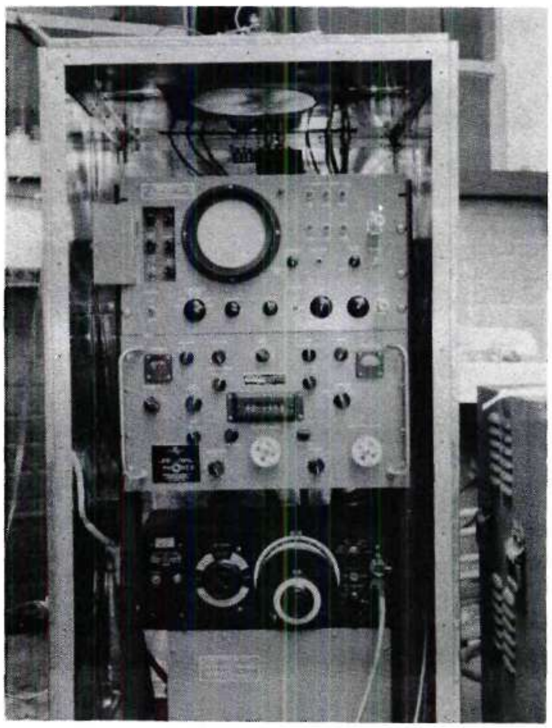
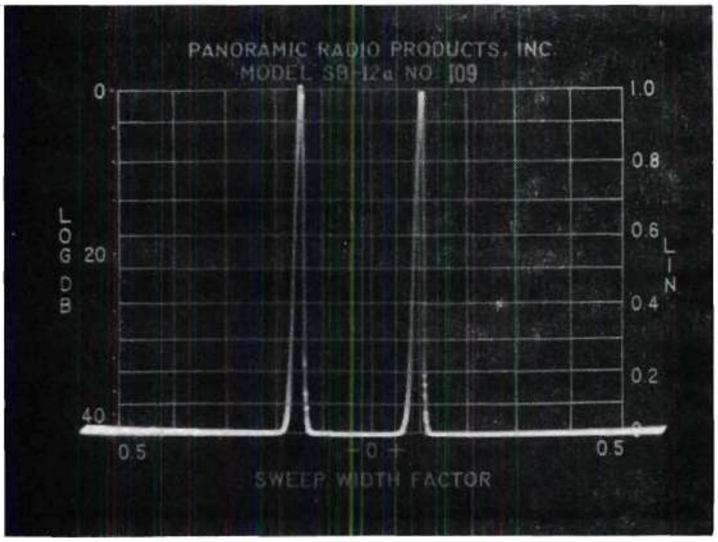


Figure 8. View of Equipment Used in the Experimental Set-Up.



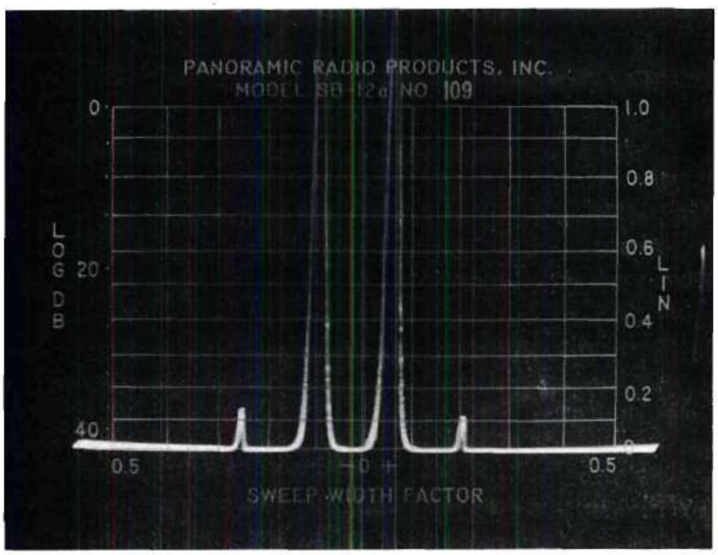


Figure 9. Spectral Distribution of Two-Tone Test Signal.

of the signal shown in these photographs is slightly more than 60 db below the level of the two tones, and the fifth order products are not detectable. This signal, with variations of its amplitude only, is the two tone test signal hereafter referred to and used throughout the experimental investigation.

The initial investigation of the beam-deflection tube began with a study of the deflection electrode to plate transfer characteristics. The experimental arrangement was essentially that of investigating the deflection electrode to plate transfer characteristic, but in particular was arranged so as to observe the difference in the plate currents flowing in the two individual anodes as a function of the absolute potential difference between the deflection electrodes. To accomplish this, the approximate operating potentials were placed on the screen grid and on the two anodes. The control grid was driven with a small ac signal and its dc potential was adjusted to such a value that the total beam current was approximately that recommended by the manufacturer. The left hand deflection electrode was ac grounded and the right hand electrode was driven by a small amplitude two tone test signal. The dc value of each deflection electrode was made adjustable with respect to ground and was arranged so that the dc difference in the deflection electrodes could be varied. With the small amplitude two tone test signal applied to the right hand deflection electrode, the average dc level of both of the deflection electrodes was varied using control R_1 and the resulting effects of the third and fifth order intermodulation products were observed and tabulated.[1] Upon completion of the run, R_1 was adjusted so as to minimize the third and fifth order intermodulation products, and the difference between the dc

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[1]Appendix II, page 56.

levels of the two electrodes was varied using control R_2 and the resulting effects were noted.[1] Both variables were then adjusted for minimum third and fifth order intermodulation product generation and a final value of third and fifth order intermodulation product rejection, with respect to the level of the desired two tones, was recorded. This procedure was repeated for each of the two[2] tubes used and in each case the individual tube was allowed to warm up for a period of five minutes.

Following this, tube No. 1 was selected and it was placed in the circuit, adjusted for minimum distortion and the levels of the distortion products were observed every fifteen minutes for a period of two and one quarter hours.[3] From this point on only tube No. 1 was investigated. The results of the various parts of this test are shown graphically in Figures 10, 11, and 12. It can be seen from the rejection levels obtained in Figure 11, that the distortion products generated are quite low for the small signal condition imposed.

The next step in the process was to determine the relation between the level of the generated intermodulation products and the level of the driving two tone test signal. For this test the tube was placed in the same circuit and both the dc level and the dc difference in the deflection electrode voltages were adjusted so as to minimize the third and fifth order intermodulation products. After this condition had been met for the

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[1] Appendix II, page 57.

[2] The initial investigation considered three tubes, but one tube was damaged and discarded. Accordingly, the data presented and conclusions drawn are based on only two tubes.

[3] Appendix II, page 57.

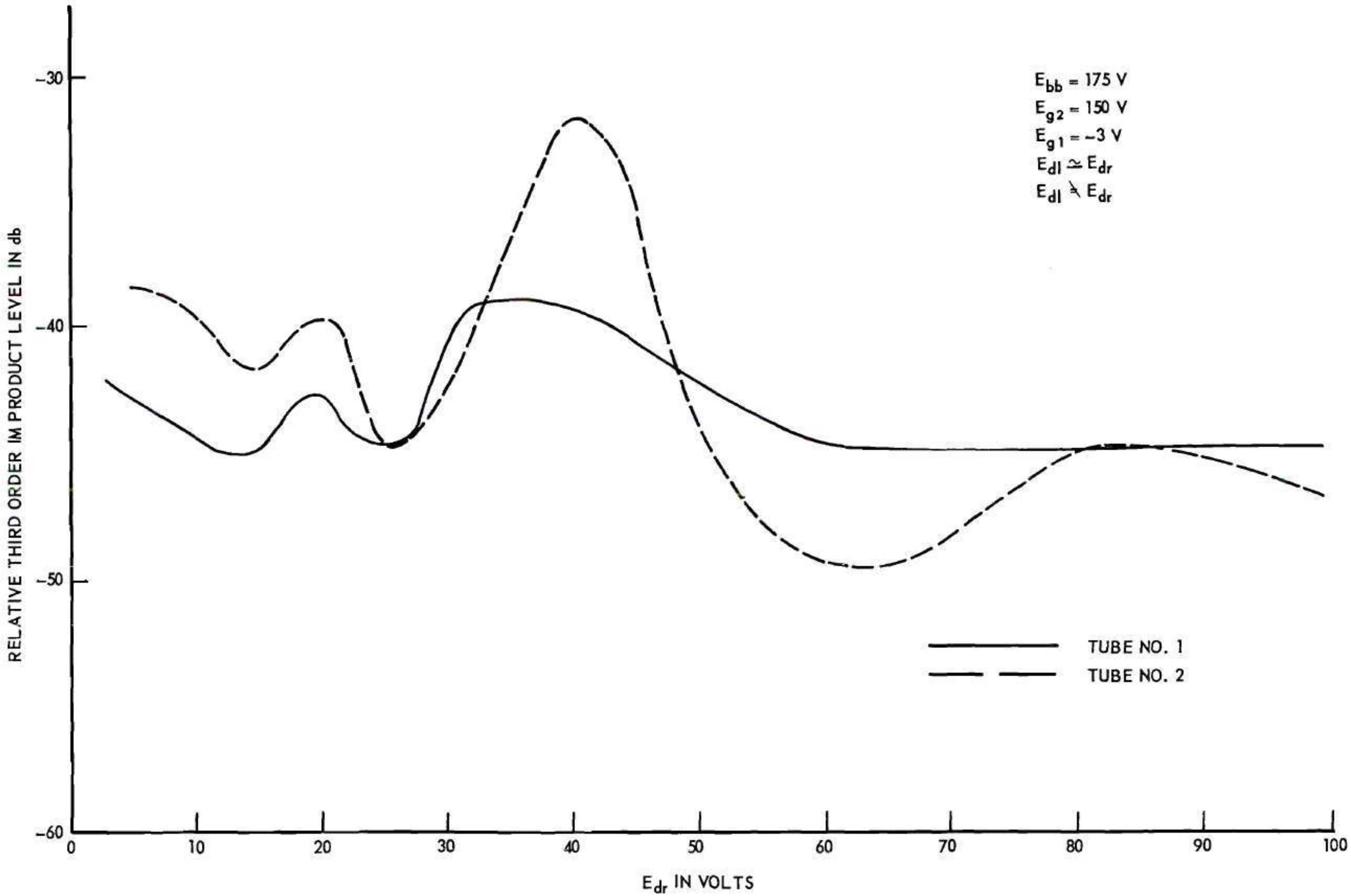

Figure 10. Relative Third Order Intermodulation Product Level Versus Deflection Electrode Potential.

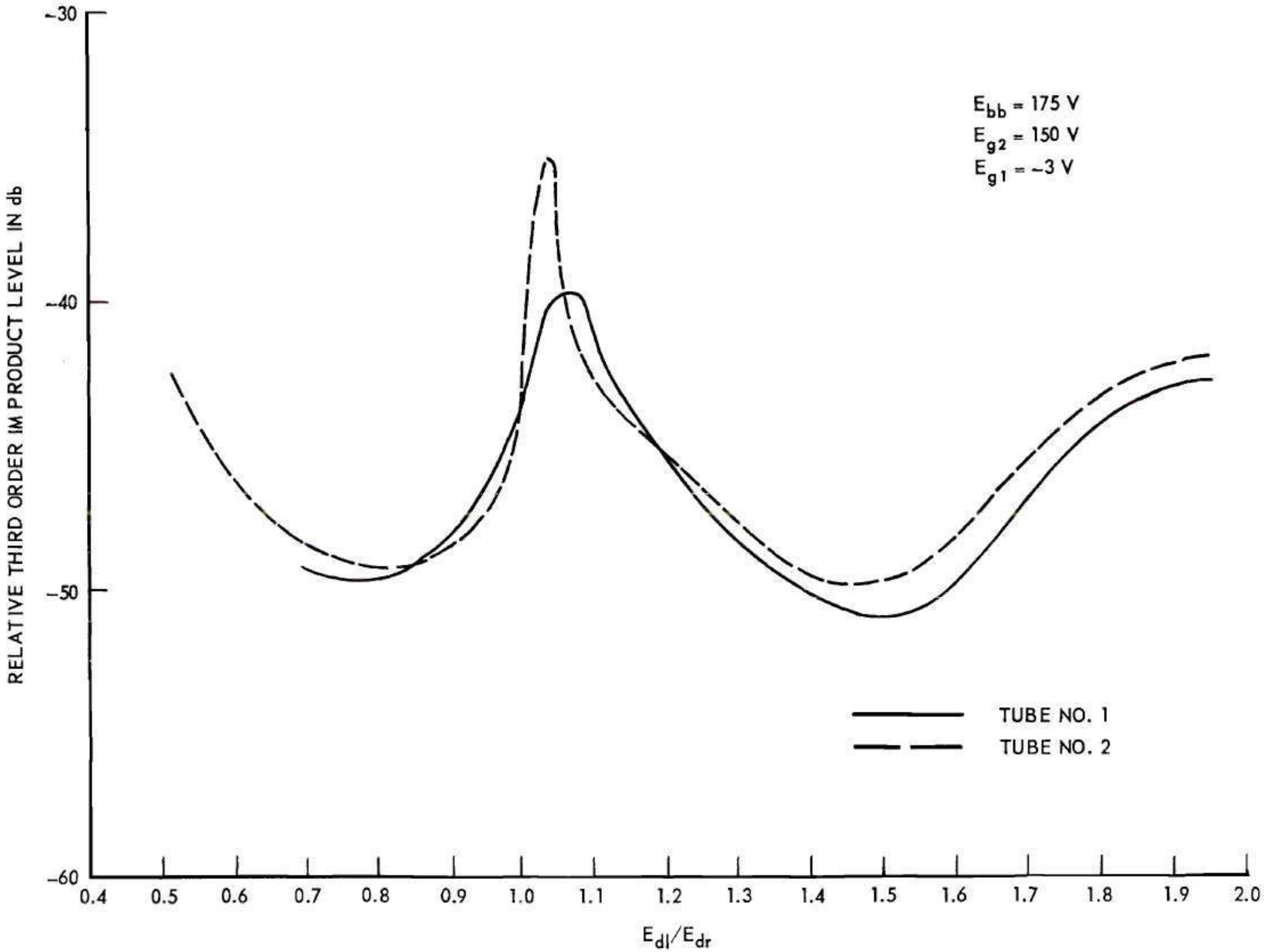

Figure 11. Relative Third Order Intermodulation Product Level Versus Ratio of Deflection Electrode Potentials.

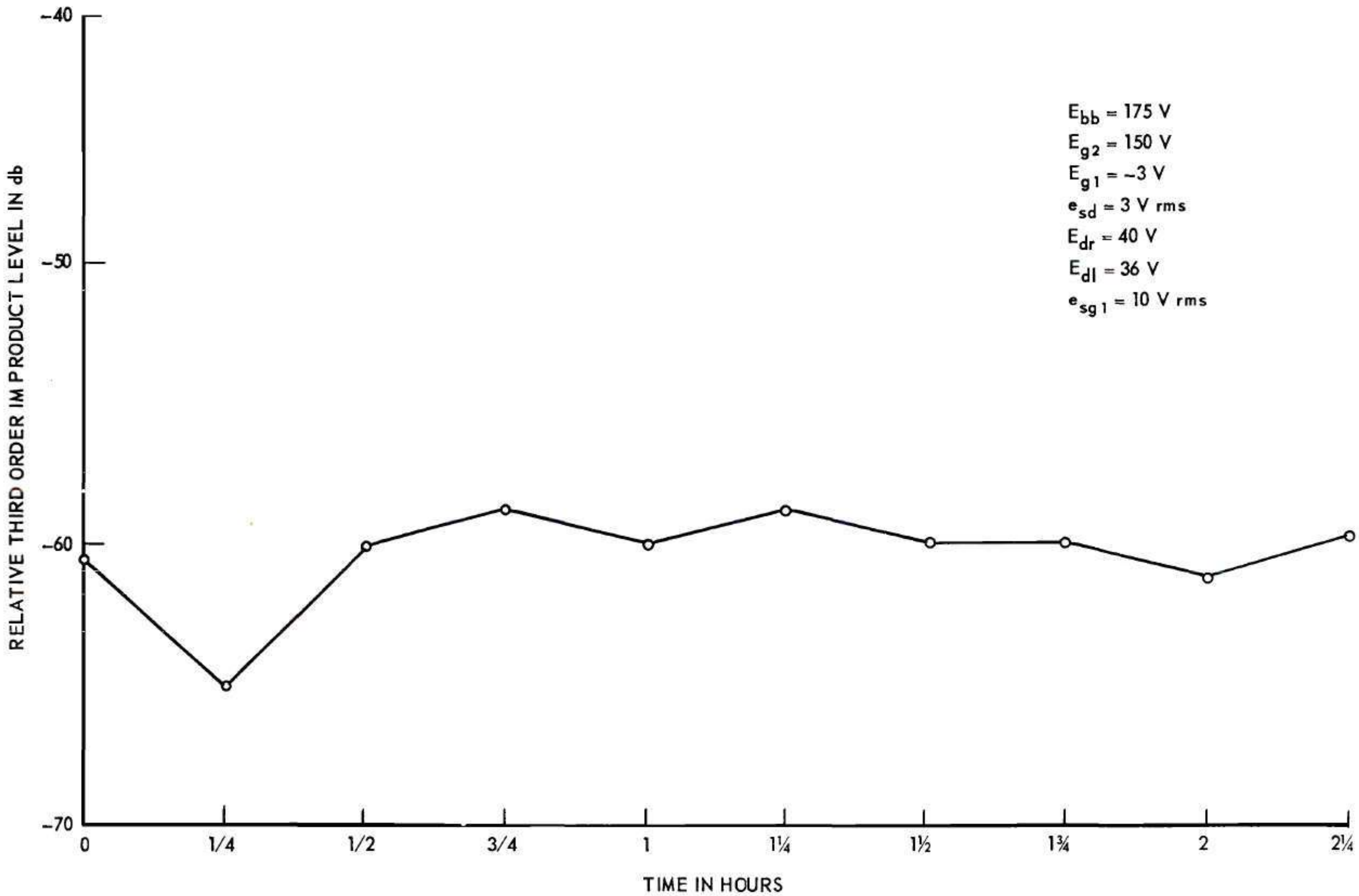

Figure 12. Third Order Intermodulation Product Rejection Versus Time.

small amplitude signal the dc levels of the deflection electrodes were not changed. The amplitude of the two tone test signal was then varied and the resulting effects were noted and data tabulated.[1] The data relating the third order IM distortion level and the two tone drive level are shown in Figure 13. After this was completed the driving signal amplitude was adjusted to a value that was not the optimum for the particular settings of the dc levels. The dc levels, both absolute and difference, of the deflection electrodes were then readjusted so as to minimize the generated distortion products and the results were noted. In connection with the latter part of this test, it was noted that within certain bounds of drive signal level, the level and difference in the dc voltage of the deflection electrodes could be adjusted so as to effect an intermodulation distortion product minimum. This suggests the possibility of a servo control that would keep the distortion products generated at a minimum. Such a control system was not implemented in the process of this work due to both time and objective limitations.

Upon the completion of these preliminary tests the effects of oscillator level were observed. A single tone signal was injected on the control grid of the tube and the two tone test signal was used to drive the deflection electrodes. The resulting output of the mixer was observed on the spectrum analyzer at the difference frequency. Both the levels of the local oscillator and the drive signal were set at an arbitrary value within the normal operating range, and the dc levels of the deflection electrodes as well as their difference voltage were adjusted so as to

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[1] Appendix II, page 58.

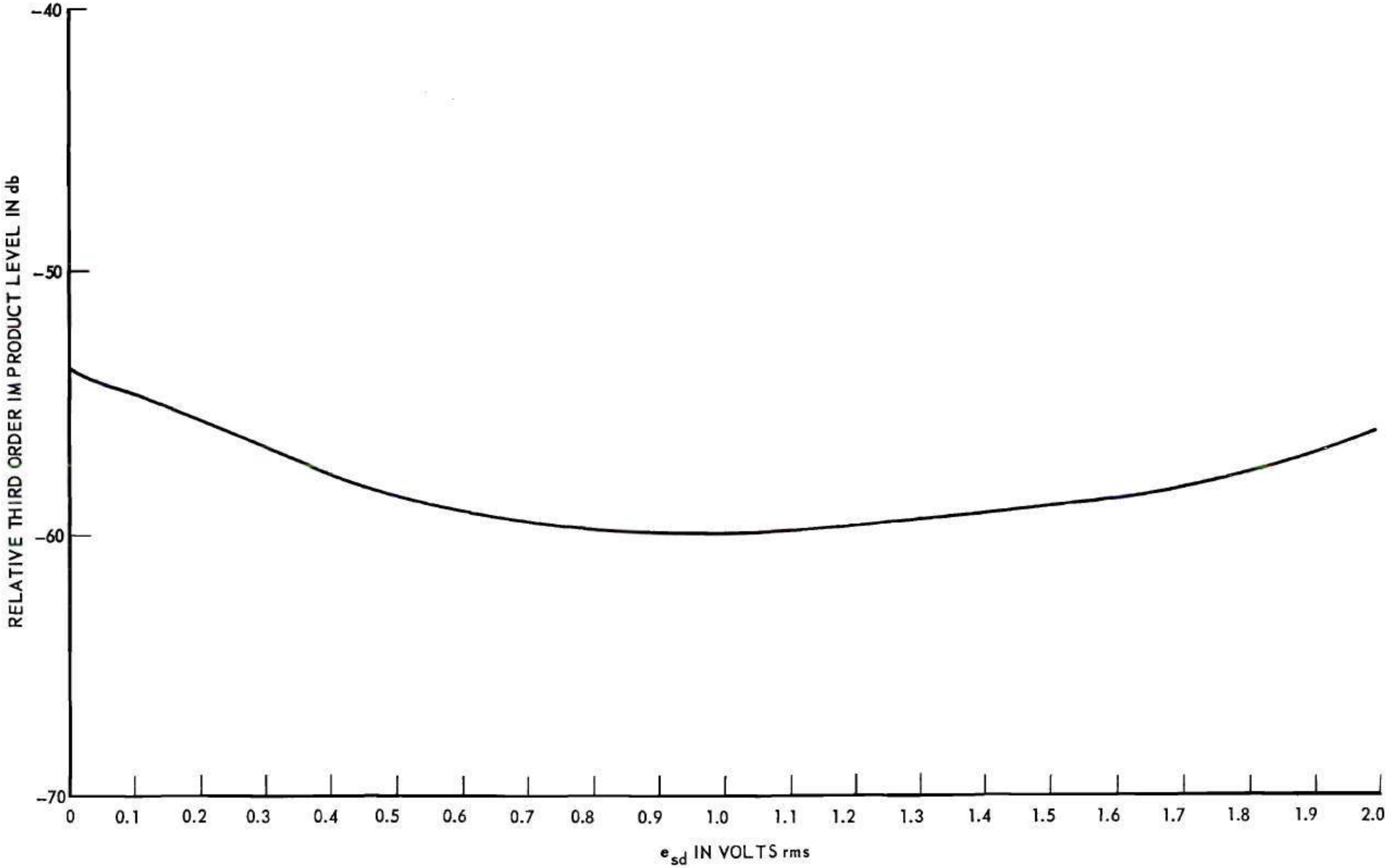

Figure 13. Relative Third Order Intermodulation Product Level Versus Level of Two-Tone Test Signal.

minimize the generated distortion products. These dc voltages then remained fixed and the level of the local oscillator signal was varied. The resulting effects on the conversion amplification and the level of the third and fifth order intermodulation products were observed and recorded.[1] The results of this test are shown graphically in Figure 14. It is interesting to note that the effect of the level of the local oscillator signal on the level of the generated intermodulation products is negligible,as long as the level is greater than three volts, as predicted by the mathematical model given in Chapter IV.

Upon completion of the preceding tests and with the previously varied parameters adjusted for maximum third and fifth order intermodulation distortion rejection data was taken establishing the relation between distortion generation and control grid bias.[2] The signal levels applied to the control grid to the deflection electrode were not varied, and only the bias applied to the control grid was varied. This test data is shown graphically in Figure 15. It will be noted that the only region of grid bias which results in an increase in intermodulation product generation is the region near cutoff bias. In this region the gain of the tube becomes very small and operation in this region would not normally be considered.

The effect on intermodulation product generation due to varying the plate supply voltage was considered next. For this test the previously fixed plate supply was replaced by a variable supply and data relating plate voltage to intermodulation product rejection were recorded.[3] This

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[1]Appendix II, page 58.

[2]Appendix II, page 59.

[3]Appendix II, page 59.

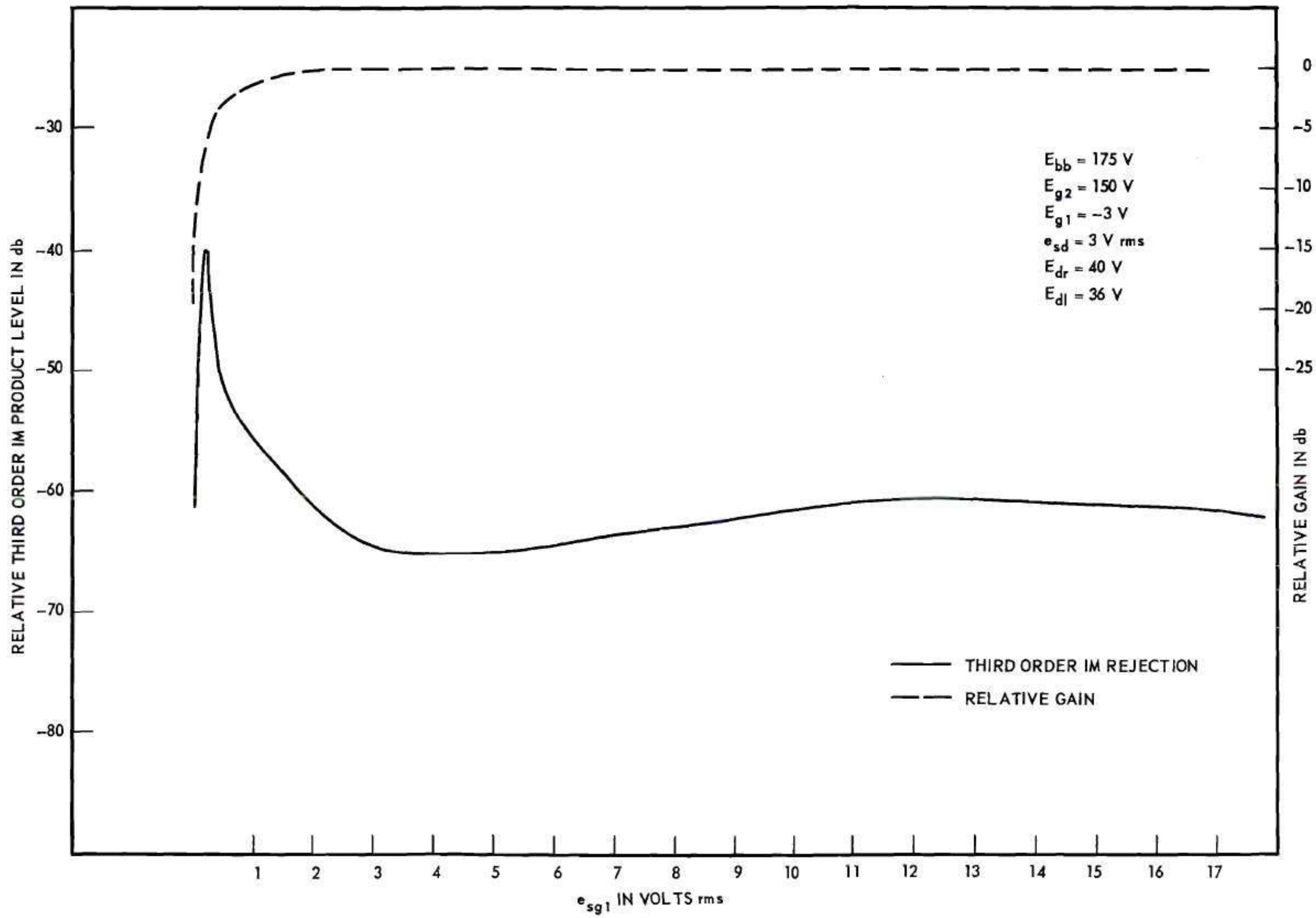

Figure 14. Relative Third Order Intermodulation Product Level and Relative Conversion Amplification Versus Level of Oscillator Injection Voltage.

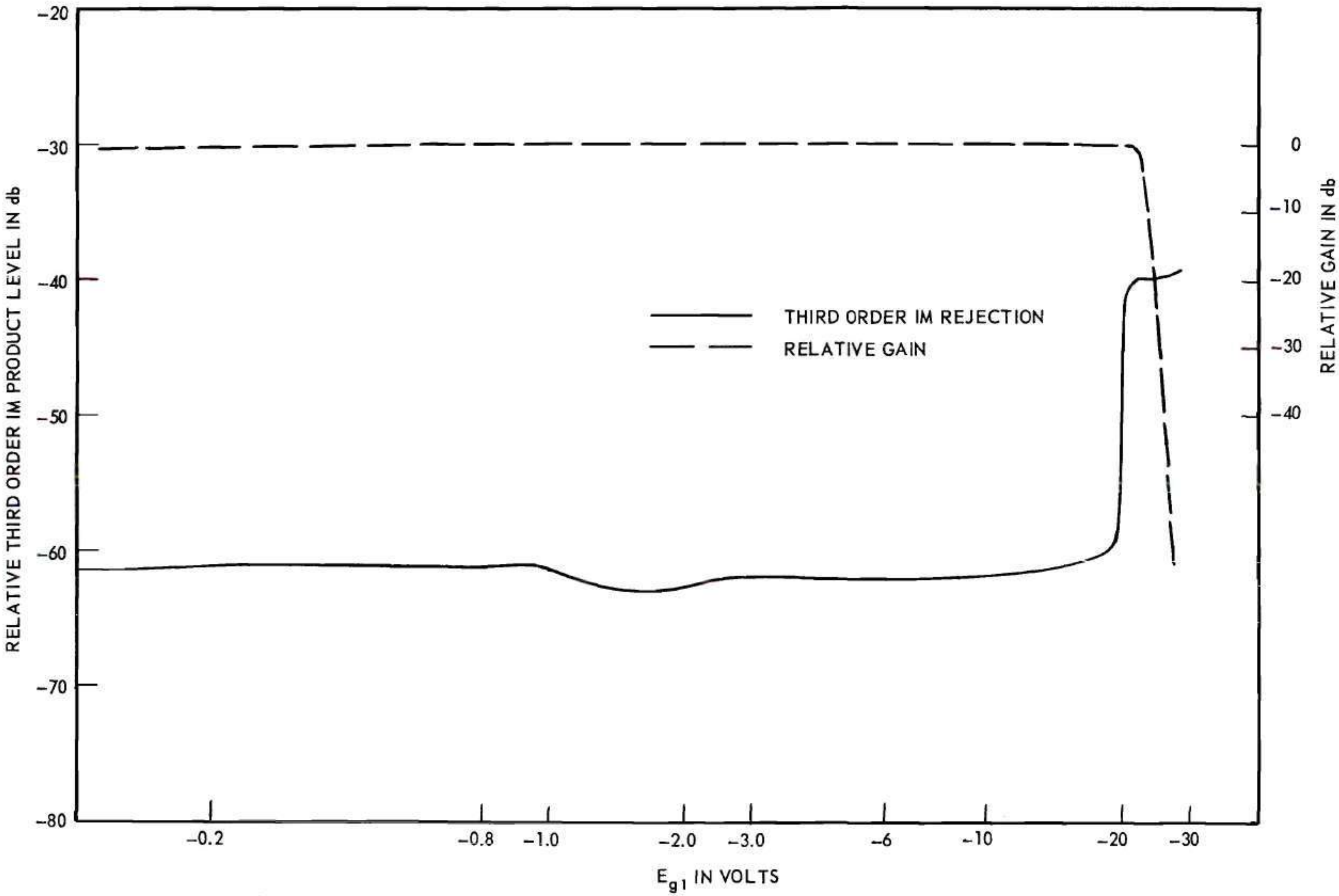

Figure 15. Relative Third Order Intermodulation Product Level and Relative Conversion Amplification Versus Control Grid Bias.

data is shown graphically in Figure 16. It is sufficient to point out here that substantial variations in the plate supply voltage do not drastically effect the relative level of the intermodulation products and the desired signal. Also plotted in the graph of Figure 16 is the relative output level of the desired signal as a function of plate voltage. In this connection it will be noted that the maximum gain occurs with any plate supply voltage between 100 and 300 volts, and throughout this range the intermodulation products are down 60 db or more.

The final test performed was designed to determine the optimum plate-to-plate impedance for low distortion operation. This was accomplished by varying the tap position of the output circuit. The output circuit presents a load of 50 ohms, and by varying the position of the tap the impedance reflected to the plate circuit was varied. Relative intermodulation product level and relative output level of the desired signal were recorded[1] and plotted in Figure 17 as a function of the tap position. The effects of varying the plate-to-plate impedance on the intermodulation products were found to be neglibiele except under conditions of very low plate impedance, but the output signal level was found to be quite dependent on this parameter as would be expected.

A tabular presentation of the data collected for all tests presented above is given in Appendix II, and a summary of the test results, recommendations, and conclusions regarding the usefulness and operation of the investigated mixer are presented in Chapter VI of this thesis.

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[1] Appendix II, page 60.

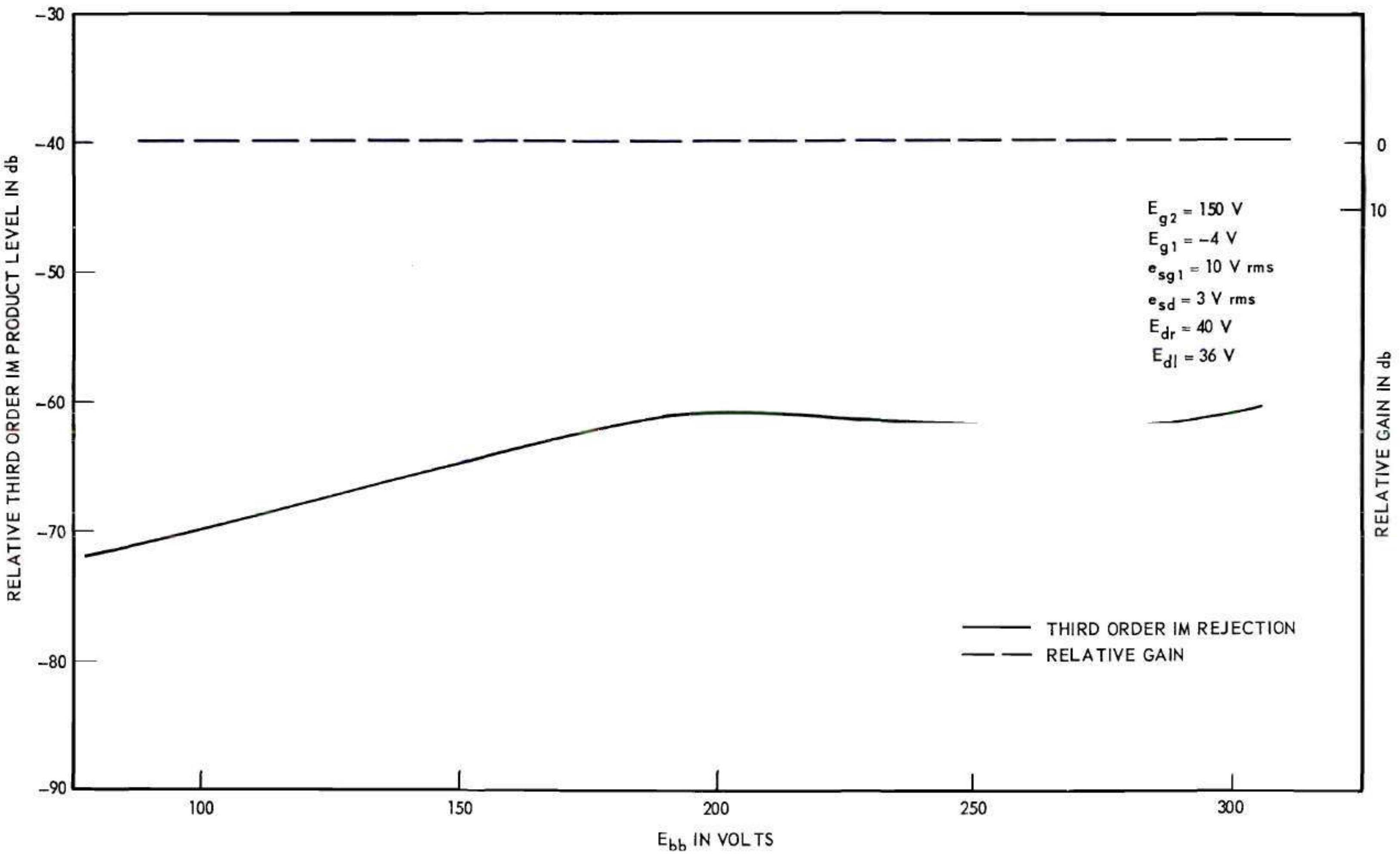

Figure 16. Relative Third Order Intermodulation Product Level and Relative Conversion Amplification Versus Plate Supply Voltage.

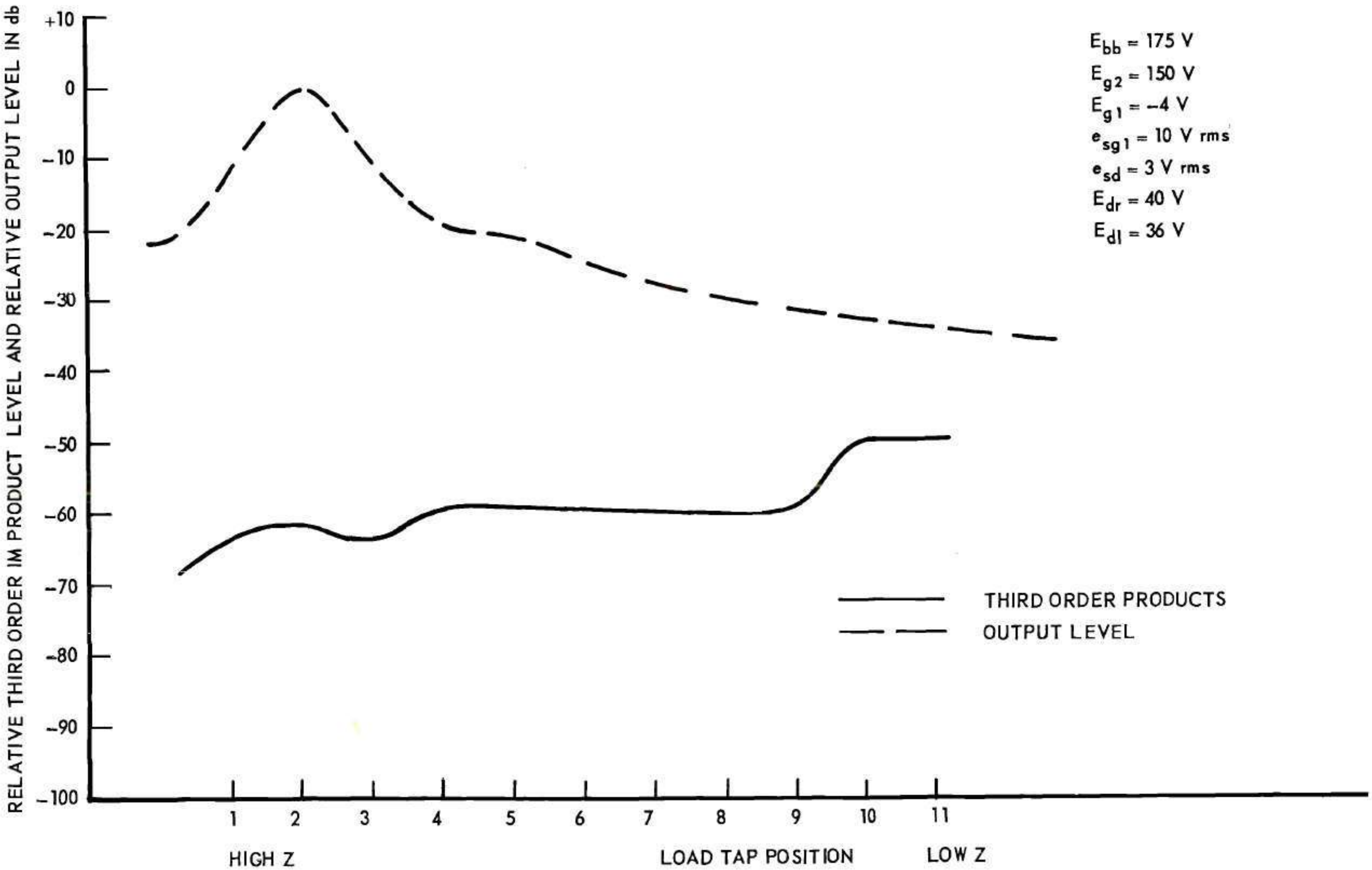

Figure 17. Relative Third Order Intermodulation Product Rejection and Relative Output Level Versus Load Tap Position.

CHAPTER VI

RESULTS OF EXPERIMENTATION, CONCLUSIONS AND RECOMMENDATIONS

The results of the experimentation indicate primarily that low distortion mixers can be designed utilizing a recently developed beam deflection tube. Specifically these results indicate that with proper design the third order intermodulation product generation can be maintained approximately 60 db below the level of the desired signal. The level to which the fifth order intermodulation products can be maintained below the desired signal level is somewhat questionable, but the results of the experimentation clearly indicate that they can in all cases be maintained below the level of the third order intermodulation products.

From a direct examination of the test results it is possible to draw the following conclusions pertaining to the specific values that should be used in the design of such a mixer which utilizes an RCA 7360 beam deflection tube as the active circuit element. The anode supply voltage should be approximately 100 volts, the screen supply voltage should be approximately 150 volts, the control grid bias voltage should be approximately -2 volts, the r-f drive voltage on the deflection electrode should be approximately 1 volt rms, and the oscillator injection voltage should be greater than 3 volts rms. The plate to plate impedance that should be presented to the tube should be approximately 4500 ohms.[1] In addition

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[1]This impedance value has been calculated from the circuit element values when the 50 ohm load was tapped in position two. This calculation is shown in Appendix I, page 55.

to the above operating parameters which should be observed, the deflection electrode potentials should be adjusted so that

$$\frac{E_{dL}}{E_{dR}} \approx 0.8 \text{ or } 1.5$$

The desirability of this operating condition can be seen by examination of the curve of Figure 11. It is important to note that in addition to the low level of intermodulation product generation that can be achieved, neither the third order intermodulation product level nor the output signal level are effected appreciably by drastic variations of oscillator injection voltage as is true in the presently used conventional mixers. In addition to the advantage of low levels of generated intermodulation products that this mixer is capable of, the conversion amplification was found to be 9 db. Thus while providing low distortion as do some other low gain mixers, this particular configuration offers an output signal amplitude advantage of 9 db.

Further investigation of low distortion mixers using this and other techniques is recommended, as well as the investigation of an electronic servo control system such as mentioned in Chapter V. Only through the efforts of the system design engineer to improve all parts of the SSB system can the full advantages and potentialities of single sideband be realized.

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4. Techniques for Linear Amplifier Tube and Circuit Design, R. E. Meek, G. H. Smith, W. M. Peacock and H. L. McKinley, Georgia Institute of Technology, Engineering Experiment Station, Final Engineering Report, Contract AF 30(602)-2146, 1961.

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APPENDICES

APPENDIX I

EQUATION DERIVATIONS

Derivation showing generation of "odd-order" or "in band" intermodulation products from Chapter II, page 5 . For convenience, Equation (5) is rewritten here as Equation (34)

$$i_p(t) = a_0 + a_1 F_{ssb}(t) + a_2 F_{ssb}(t)^2 + a_3 F_{ssb}(t)^3 + a_4 F_{ssb}(t)^4 + \ldots\ldots + a_n F_{ssb}(t)^n + \ldots. \tag{34}$$

where $F_{ssb}(t)$ is a single sideband input signal. It has been found that a two tone signal adequately simulates single sideband operation,[1] and such a signal is given below:

$$F_{ssb}(t) = A \cos \omega_1 t + A \cos \omega_2 t = \text{input signal} \tag{35}$$

Then

$$\begin{aligned} F_{ssb}(t)^2 &= A^2 \cos^2 \omega_1 t + 2A^2 \cos \omega_1 t \cos \omega_2 t + A^2 \cos^2 \omega_2 t \qquad (36) \\ &= A^2 \left(\tfrac{1}{2} + \tfrac{1}{2} \cos 2\omega_1 t\right) + A^2 \left(\tfrac{1}{2} + \tfrac{1}{2} \cos 2\omega_2 t\right) \\ &\quad + 2A^2 \left[\tfrac{1}{2} \cos (\omega_1 - \omega_2)t + \tfrac{1}{2} \cos (\omega_1 + \omega_2)t\right] \\ &= A^2 + A^2 \cos (\omega_1 - \omega_2)t + A^2 \cos (\omega_1 + \omega_2)t \\ &\quad + \frac{A^2}{2} \cos 2\omega_1 t + \frac{A^2}{2} \cos 2\omega_2 t \end{aligned}$$

$$
\begin{aligned}
F_{ssb}(t)^3 = {} & A^3 \cos \omega_1 t + A^3 \cos \omega_1 t \cos (\omega_1 - \omega_2)t \\
& + A^3 \cos \omega_1 t \, (\omega_1 + \omega_2)t + \frac{A^3}{2} \cos \omega_1 t \cos 2\omega_2 t \\
& + \frac{A^3}{2} \cos \omega_1 t \cos 2\omega_2 t + A^3 \cos \omega_2 t \\
& + A^3 \cos \omega_2 t \cos (\omega_1 - \omega_2)t + A^3 \cos \omega_2 t \cos (\omega_1 + \omega_2)t \\
& + \frac{A^3}{2} \cos \omega_2 t \cos 2\omega_1 t + \frac{A^3}{2} \cos \omega_2 t \cos 2\omega_2 t \\
= {} & A^3 \cos \omega_1 t + A^3 \left[\frac{1}{2} \cos (\omega_2)t + \frac{1}{2} \cos (2\omega_1 - \omega_2)t\right] \\
& + A^3 \left[\frac{1}{2} \cos (-\omega_2)t + \frac{1}{2} \cos (2\omega_1 + \omega_2)t\right] \\
& + \frac{A^3}{2} \left[\frac{1}{2} \cos (-\omega_1)t + \frac{1}{2} \cos (3\omega_1)t\right] + \frac{A^3}{2} \left[\frac{1}{2} \cos (\omega_1 \right. \\
& \left. - 2\omega_2)t + \frac{1}{2} \cos (\omega_1 + 2\omega_2)t\right] + A^3 \cos \omega_2 t \\
& + A^3 \left[\frac{1}{2} \cos (2\omega_2 - \omega_1)t\right] + \frac{1}{2} \cos (\omega_1)t + A^3 \left[\frac{1}{2} \cos (\omega_1)t \right. \\
& \left. + \frac{1}{2} \cos (\omega_1 + 2\omega_2)t\right] + \frac{A^3}{2} \left[\frac{1}{2} \cos (\omega_2 - 2\omega_1)t \right. \\
& \left. + \frac{1}{2} \cos (\omega_2 + 2\omega_1)t\right] + \frac{A^3}{2} \left[\frac{1}{2} \cos (-\omega_2)t \right. \\
& \left. + \frac{1}{2} \cos (3\omega_2)t\right]
\end{aligned}
\tag{37}
$$

Substituting Equations (35), (36), and (37) into Equation (34) we find the output current to be

$$
\begin{aligned}
i_p(t) = {} & a_0 + a_1 \left[A \cos \omega_1 t + A \cos \omega_2 t\right] + a_2 \Big[A^2 \qquad (38)\\
& + A^2 \cos (\omega_1 - \omega_2)t + A^2 \cos (\omega_1 + \omega_2)t + \frac{A^2}{2} \cos 2\omega_1 t\\
& + \frac{A^2}{2} \cos 2\omega_2 t\Big] + a_3 \Big[A^3 \cos \omega_1 t + \frac{A^3}{2} \cos \omega_2 t\\
& + \frac{A^3}{2} \cos (2\omega_1 - \omega_2)t\Big] + \frac{A^3}{2} \cos \omega_2 t\\
& + \frac{A^3}{3} \cos (2\omega_1 + \omega_2)t + \frac{A^3}{4} \cos \omega_1 t + \frac{A^3}{4} \cos 3\omega_1 t\\
& + \frac{A^3}{4} \cos (2\omega_2 - \omega_1)t + \frac{A^3}{4} \cos (2\omega_2 + \omega_1)t + A^3 \cos \omega_2 t\\
& + \frac{A^3}{2} \cos (2\omega_2 - \omega_1)t + \frac{A^3}{2} \cos \omega_1 t\\
& + \frac{A^3}{2} \cos (2\omega_3 + \omega_1)t + \frac{A^3}{4} \cos (2\omega_1 - \omega_2)t\\
& + \frac{A^3}{4} \cos (2\omega_1 + \omega_2)t + \frac{A^3}{4} \cos \omega_2 t + \frac{A^3}{4} \cos 3\omega_2 t
\end{aligned}
$$

+ contributions from fifth and higher order curvature.

If we now consider the current components of Equation (38) which fall on or near the original signal frequency we will be considering the components which are in the frequency band of interest, thus the name "in band" distortion products. In order to distinguish which components are near the original signal frequency we must have some notion as to the relative order of magnitude of ω_1 and ω_2. The average frequency of the input signal is $\frac{\omega_1+\omega_2}{2}$ and we may restrict the conditions on ω_1 and ω_2 so that $\omega_2-\omega_1 << \frac{\omega_1+\omega_2}{2}$; where $\omega_2 > \omega_1$. The resulting inband current components, $i_{p_{IB}}(t)$, are given below.

$$i_{p_{IB}}(t) = a_1 A \cos \omega_1 t + a_1 A \cos \omega_2 t + a_3 A^3 \cos \omega_1 t \qquad (39)$$

$$+ \frac{a_3 A^3}{2} \cos \omega_2 t + \frac{a_3 A^3}{2} \cos (2\omega_1 - \omega_2)t + \frac{a_3 A^3}{2} \cos \omega_2 t$$

$$+ \frac{a_3 A^3}{4} \cos \omega_1 t + \frac{a_3 A^3}{4} \cos (2\omega_2 - \omega_1)t + a_3 A^3 \cos \omega_2 t$$

$$+ \frac{a_3 A^3}{2} \cos (2\omega_2 - \omega_1)t + \frac{a_3 A^3}{2} \cos \omega_1 t + \frac{a_3 A^3}{2} \cos \omega_1 t$$

$$+ \frac{a_3 A^3}{4} \cos (2\omega_1 - \omega_2)t + \frac{a_3 A^3}{4} \cos \omega_2 t$$

+ contributions from higher order terms.

Collecting components of the same frequency we have,

$$i_{p_{IB}}(t) = K_1 \cos \omega_1 t + K_2 \cos \omega_2 t \qquad (40)$$

$$+ K_3 \cos (2\omega_2 - \omega_1)t + K_4 \cos (2\omega_1 - \omega_2)t$$

where

$$K_1 = K_2 = \frac{4a_1 A + 9a_3 A^3}{4} \qquad (41)$$

and

$$K_3 = K_4 = \frac{3a_3 A^3}{4} \qquad (42)$$

From Equation (40) it can be seen that the only frequency components, other than the two original frequencies, are of the form $\cos \left[P\omega_1 - (P-1)\omega_2 \right] t$ and $\cos \left[P\omega_2 - (P-1)\omega_1 \right] t$. The number $P + (P - 1)$ is always odd and thus accounts for the name "odd order" intermodulation product. It will also be noticed that $P \geq 2$ and will always be an integer, for if P were equal to one the component would merely be one of the original two tones.

Expansion of Equation 27, page 20.

Rewriting Equation (27) we have,

$$i_d = c_0 + c_1 e_{LO} + c_2 e_{LO}^2 + c_3 e_{LO}^3 + c_4 e_{LO}^4 + c_5 e_{LO}^5 + \ldots\ldots \quad (42)$$

$$+ D_1 e_s + D_2 e_{LO} e_s + D_3 e_{LO}^2 e_s + D_4 e_{LO}^3 e_s + \ldots\ldots$$

$$i_d = c_0 + D_1 e_s + e_{LO}(c_1 + D_2 e_s) + e_{LO}^2 (c_2 + D_3 e_s) \quad (43)$$

$$+ e_{LO}^3 (c_3 + D_4 e_s) + e_{LO}^4 (c_4 + D_5 e_s) +$$

$$+ e_{LO}^5 + (c_5 + D_6 e_s) + \ldots\ldots$$

Substituting equations (29) and (30) into Equation (43) we have,

$$i_d = c_0 + D_1 \left[B \cos \omega_2 t + B \cos \omega_3 t\right] + A \cos \omega_1 t \left[c_1\right. \quad (44)$$

$$\left. + D_2 \, (B \cos \omega_2 t + B \cos \omega_3 t)\right] + A^2 \cos^2 \omega_1 t \left[c_2\right.$$

$$\left. + D_3 \, (B \cos \omega_2 t + B \cos \omega_3 t)\right] + A^3 \cos^3 \omega_1 t \left[c_3\right.$$

$$\left. + D_4 \, (B \cos \omega_2 t + B \cos \omega_3 t)\right] + A^4 \cos^4 \omega_1 t \left[c_4\right.$$

$$\left. + D_5 \, (B \cos \omega_2 t + B \cos \omega_3 t)\right] + \ldots$$

but

$$e_{LO}^{\ 2} = A^2 \cos^2 \omega_1 t = \frac{A^2}{2} \left[\cos 2\omega_1 t + 1\right] \tag{45}$$

$$e_{LO}^{\ 3} = A^3 \cos^3 \omega_1 t = \frac{A^3}{2} \left[\cos 2\omega_1 t \cos \omega_1 t + \cos \omega_1 t\right] \tag{46}$$

$$= \frac{A^3}{2} \left[\frac{1}{2} (\cos 2\omega_1 t + \cos \omega_1 t) + \cos \omega_1 t\right]$$

$$= \frac{A^3}{4} \left[\cos 3\omega_1 t + 3 \cos \omega_1 t\right]$$

$$e_{LO}^{\ 4} = A^4 \cos^4 \omega_1 t = \frac{A^4}{4} \left[\cos 3\omega_1 t + 3 \cos \omega_1 t\right] \cos \omega_1 t \tag{47}$$

$$= \frac{A^4}{4} \left[\frac{1}{2} (\cos 4\omega_1 t + \cos 2\omega_1 t + 3 \cos 2\omega_1 t + 3)\right]$$

$$= \frac{A^4}{8} \left[\cos 4\omega_1 t + 4 \cos 2\omega_1 t + 3\right]$$

$$e_{LO}^{\ 5} = A^5 \cos^5 \omega_1 t = \frac{A^4}{8} \left[\cos 4\omega_1 t + 4 \cos 2\omega_1 t + 3\right] A \cos \omega_1 t \tag{48}$$

$$= \frac{A^5}{8} \left[\frac{1}{2} (\cos 3\omega_1 t + \cos 5\omega_1 t) + 2(\cos \omega_1 t\right.$$

$$\left. + \cos 3\omega_1 t) + 3 \cos \omega_1 t\right]$$

$$= \frac{A^5}{16} \left[\cos 3\omega_1 t + \cos 5\omega_1 t + 4 \cos \omega_1 t\right.$$

$$\left. + 4 \cos 3\omega_1 t + 6 \cos \omega_1 t\right]$$

$$= \frac{A^5}{16} \left[\cos 5\omega_1 t + 5 \cos 3\omega_1 t + 10 \cos \omega_1 t\right]$$

Rewriting Equation (44) and substituting in Equations (45), (46), (47), and (48) we have

$$i_d = C_0 + D_1(B \cos \omega_2 t + B \cos \omega_3 t) + A \cos \omega_1 t \Big[C_1 + D_2 (B \cos \omega_2 t + B \cos \omega_3 t)\Big] + \frac{A^2}{2} \Big[\cos 2\omega_1 t + 1\Big]\Big[C_2 + D_3 (B \cos \omega_2 t + B \cos \omega_3 t)\Big] + \frac{A^3}{4} \Big[\cos 3\omega_1 t + 3 \cos \omega_1 t\Big] \cdot \Big[C_3 + D_4(B \cos \omega_2 t + B \cos \omega_3 t)\Big] + \frac{A^4}{8} \Big[\cos 4\omega_1 t + 4 \cos 2\omega_1 t + 3\Big]\Big[C_4 + D_5 (B \cos \omega_2 t + B \cos \omega_3 t)\Big] + \frac{A^5}{16} \Big[\cos 5\omega_1 t + 5 \cos 3\omega_1 t + 10 \cos \omega_1 t\Big] + \Big[C_5 + D_6(B \cos \omega_2 t + B \cos \omega_3 t)\Big] + \ldots \tag{49}$$

Collecting terms and expanding we have,

$$\begin{aligned} i_d = C_0 &+ \cos \omega_1 t \left[C_1 A + \frac{3A^3 C_3}{4} + \frac{5A^5 C_5}{8} + \ldots. \right] \\ &+ \cos 2\omega_1 t \left[\frac{A^2 C_2}{2} + \frac{A^4 C_5}{2} + \ldots. \right] \\ &+ \cos 3\omega_1 t \left[\frac{A^3 C_3}{4} + \frac{5A^5 C_5}{16} + \ldots. \right] \\ &+ \cos 4\omega_1 t \left[\frac{A^4 C_4}{8} + \ldots\right] \\ &+ \cos \omega_2 t \left[BD_1 + \frac{A^2 D_3 B}{2} + \frac{3A^4 D_5 B}{8} + \ldots \right] \\ &+ \cos \omega_3 t \left[BD_1 + \frac{A^2 D_3 B}{2} + \frac{3A^4 D_5 B}{8} + \ldots \right] \end{aligned} \tag{50}$$

$$+ \cos(\omega_2-\omega_1)t \left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(\omega_3-\omega_1)t \left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(\omega_2+\omega_1)t \left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(\omega_3+\omega_1)t \left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(2\omega_1-\omega_2)t \left[\frac{A^2D_3B}{4} + \frac{A^4D_5B}{4} + \ldots\right]$$

$$+ \cos(2\omega_1-\omega_3)t \left[\frac{A^2D_3B}{4} + \frac{A^4D_5B}{4} + \ldots\right]$$

$$+ \cos(2\omega_1+\omega_2)t \left[\frac{A^2D_3B}{4} + \frac{A^4D_5B}{4} + \ldots\right]$$

$$+ \cos(2\omega_1+\omega_3)t \left[\frac{A^2D_3B}{4} + \frac{A^4D_5B}{4} + \ldots\right]$$

$$+ \cos(3\omega_1-\omega_2)t \left[\frac{A^3D_4B}{8} + \frac{5A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(3\omega_1-\omega_3)t \left[\frac{A^3D_4B}{8} + \frac{5A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(3\omega_1+\omega_2)t \left[\frac{A^3D_4B}{8} + \frac{5A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(3\omega_1+\omega_3)t \left[\frac{A^3D_4B}{8} + \frac{5A^5D_6B}{32} + \ldots\right]$$

$$+ \cos(4\omega_1-\omega_2)t \left[\frac{A^4D_5B}{16} + \ldots\right]$$

$$+ \cos (4\omega_1-\omega_3)t \left[\frac{A^4D_5B}{16} + \dots\right]$$

$$+ \cos (4\omega_1+\omega_2)t \left[\frac{A^4D_5B}{16} + \dots\right]$$

$$+ \cos (4\omega_1+\omega_3)t \left[\frac{A^4D_5B}{16} + \dots\right]$$

$$+ \cos (5\omega_1-\omega_2)t \left[\frac{A^5D_6B}{32} + \dots\right]$$

$$+ \cos (5\omega_1-\omega_3)t \left[\frac{A^5D_6B}{32} + \dots\right]$$

$$+ \cos (5\omega_1+\omega_2)t \left[\frac{A^5D_6B}{32} + \dots\right]$$

$$+ \cos (5\omega_1+\omega_3)t \left[\frac{A^5D_6B}{32} + \dots\right]$$

$$+ \frac{C_2A^2}{2} + \frac{3C_4A^4}{8}$$

When the output is ac coupled and tuned to the difference frequency we have

$$i_{IF} = K \left\{ \cos (\omega_2-\omega_1)t \left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \dots\right] \right. \tag{51}$$

$$\left. + \cos (\omega_3 - \omega_1)t\left[\frac{AD_2B}{2} + \frac{3A^3D_4B}{8} + \frac{10A^5D_6B}{32} + \dots\right]\right\}$$

where K = const

The next adjacent output frequency components, $2\omega_1-\omega_2$ and $2\omega_1-\omega_3$ fall far enough away from the desired output so that they can be filtered out without difficulty if ω_1 is chosen correctly.

Derivation of desired plate impedance.

From Figure 17 it can be seen that the optimum tap position is $\frac{2}{11}$ of the secondary winding when 50 Ω is the load. The transformer turns ratio was measured and found to be 1:1.28. Accordingly, the impedance reflected to the tube is Z_0 where

$$Z_0 = \left[\left(\frac{11}{2}\right) (50) \quad (1.28) \right]^2 = 4500 \text{ ohms.}$$

APPENDIX II

TABULATED TEST DATA

Test No. 1, page 28.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -3 volts. A small signal was applied to the deflection electrode and the following data were collected.

		Tube No. 1		Tube No. 2	
E_{dL} Volts	E_{dr} Volts	3rd Order in db	5th Order in db	3rd Order in db	5th Order in db
115	100	-45	-	-47	-
95	80	-45	-	-45	-
66	60	-45	-	-45	-
45	40	-45	-	-50	-
33	30	-40	-	-42	-
27	25	-45	-	-45	-
23	20	-43	-	-40	-
17	15	-45	-	-42	-
12	10	-45	-	-40	-
0	0	-42	-	-	-

Notes: Drive level was 1.4 volts rms on the deflection electrode with a 60:1 step up ratio in the resonate transformer. In this and the following data tables the dashes associated with the fifth order intermodulation products indicate that the fifth order IM products were not detectable.

Test No. 2, page 29.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -3 volts. The average value of E_{dL} and E_{dr}, $\frac{E_{dr} + E_{dL}}{2}$, was adjusted to minimize the 3rd and 5th Order IM products and the relative values of E_{dr} and E_{dL} were varied.

E_{dL}/E_{dr}	E_{dL} Volts	E_{dr} Volts	3rd Order in db	5th Order in db	3rd Order in db	5th Order in db
1.08	27	25	-45	-	-	-
1.1	33	30	-40	-	-	-
1.185	32	27	-45	-	-	-
1.61	37	23	-50	-	-	-
1.77	39	22	-45	-	-	-
0.58	5.5	9.5	-	-	-45	-
1.06	37	35	-	-	-35	-
1.08	27	25	-	-	-45	-
1.1	33	30	-	-	-42	-
1.45	45	31	-	-	-35	-
1.53	55	37.5	-	-	-55	-

Test No. 3, page 29.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -3 volts, e_{sd} = 3 volts rms, E_{dr} = 40.0 volts, E_{dL} = 36.0 volts, e_{shl} = 10 volts rms

Time in hours	3rd Order	5th Order
0	-60	-
0.25	-65	-
0.50	-60	-
0.75	-59	-
1.0	-60	-
1.25	-59	-
1.50	-60	-
1.75	-60	-
2.0	-61	-
2.25	-60	-

Test No. 4, page 33.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -3 volts, E_{dr} = 9.0 volts, E_{dL} = 24.5 volts.

Adjust the average of E_{dr} and E_{dL} and the values of E_{dr} and E_{dL} to minimize the third and fifth order IM products. The level of the two tone drive signal was varied and the following data collected.

Tube No. 1

e_{sd} Volts	3rd Order in db	5th Order in db
0.12	-55	-
0.25	-56	-
0.8	-60	-
1.3	-59	-
1.9	-57	-
3.0	-50	-

Test No. 5, page 35.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -3 volts, E_{dL} = 36.0 volts
E_{dr} = 40.0 volts, e_{sd} = 3 volts rms,

e_{sg1} Volts rms	3rd Order in db	5th Order in db	Desired Signal Level Relative to Maximum in db
0.1	-60	-	-19
0.2	-40	-	-7
0.5	-50	-	-3
1.0	-55	-	-3
2.0	-61	-	-1
3.0	-65	-	0
10.0	-62	-	0
15.0	-62	-	0

Test No. 6, page 35.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{dL} = 36.0 volts, E_{dr} = 40.0 volts

e_{sd} = 3 volts rms, e_{sg1} = 10.0 volts rms

E_{g1} Volts	3rd Order in db	5th Order in db	Desired Signal Level Relative to Maximum in db
0	-61	-	0
-1	-61	-	0
-2	-63	-	0
-3	-62	-	0
-4	-62	-	0
-5	-62	-	0
-10	-62	-	0
-20	-60	-	0
-22.5	-40	-	0
-25	-40	-	-40

Test No. 7, page 35.

E_{g2} = 150 volts, E_{g1} = 4 volts, E_{dL} = 36 volts, E_{dr} = 40 volts

E_{sd} = 3 volts rms, e_{sg1} = 10 volts rms

E_{bb} Volts	3rd Order in db	5th Order in db	Desired Signal Level Relative to Maximum in db
300	-61	-	0
250	-62	-	0
200	-61	-	0
150	-65	-	0
100	-70	-	0

Test No. 8, page 38.

E_{bb} = 175 volts, E_{g2} = 150 volts, E_{g1} = -4 volts, E_{dL} = 36 volts, E_{dr} = 40 volts E_{sd} = 3 volts rms, and e_{sg1} = 10 volts rms

Impedance Tap Point Volts	3rd Order in db	5th Order in db	Desired Signal Level Relative to Maximum in db
1	-65	-	-12
2	-62	-	0
3	-65	-	-12
4	-60	-	-20
5	-60	-	-21
6	-60	-	-25
7	-60	-	-28
8	-60	-	-30
9	-60	-	-32
10	-50	-	-33
11	-50	-	-35